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AMPLITUDE EFFECTS ON THE DYNAMIC PERFORMANCE OF A HYDROSTATIC GAS THRUST BEARING

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Amplitude Effects on the Dynamic Performance of a Hydrostatic Gas Thrust Bearing

by

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ABSTRACT

The Reynolds' equation is applied to a strip gas thrust bearing to analyze amplitude disturbance effects on its dynamic performance. The Reynolds' equation, a nonlinear time-dependent parabolic partial differential equation is numerically approximated using finite-difference techniques. The time dependent load carrying capacity is represented by a Fourier series up to and including the third harmonics. Design curves for the load capacity and the linear stiffness and damping are presented as a function of inlet location, restrictor coefficient, supply pressure, amplitude of oscillation, and squeeze number. For the range of amplitudes investigated the dimensionless load capacity, stiffness and damping do not exhibit an appreciable change in magnitude; thus, only one design curve is needed to represent each relationship. A design methodology is presented.

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NOMENCLATURE

SYMBOL	DESCRIPTION
*	Denotes real variable
a	Dimensionless distance from centerline (x=0) to the inlets
A_n, B_n	Fourier coefficients (n=1,2,3,....)
C_D	Orifice discharge coefficient
D*	Damping
D	Dimensionless damping $(D^*(1-\varepsilon^2)^{1.5}/2\mu L^* (\frac{w^*}{h_o^*})^3)$
g_o	Gravitational acceleration
h*	Film thickness
h_o^*	Mean film thickness
h	Dimensionless film thickness (h^*/h_o^*)
i	Spatial node index
k	Ratio of specific heats (C_p/C_V)
K*	Stiffness
K_s	Dimensionless stiffness $(K^*h_o^*/2w^*L^*P_a^*(P_s-1))$
L*	Bearing length
M_I^*	Mass flow from central region
M_{II}^*	Mass flow through the sill area
M_R^*	Mass flow through restrictor
M_o, M_{RO}	Mean mass flow
MAX	Maximum number of nodes
n	Time node index

NOMENCLATURE (continued)

SYMBOL	DESCRIPTION
P^*	Pressure
P	Dimensionless pressure ($P^*/P_a{}^*$)
$P_a{}^*$	Ambient pressure
P_o	Dimensionless static pressure downstream of inlet
P_s	Supply pressure
r	Iteration counter
R	Gas constant
s	Iteration counter
t^*	Time
t	Dimensionless time ($t^*\Omega^*$)
T	Gas temperature
w^*	Bearing width
W^*	Total bearing load
W	Dimensionless bearing load ($W^*/2L^*w^*P_a{}^*$)
$\overline{W}$	Average load capacity
$\overline{W}_o$	Normalized average load capacity ($\overline{W}/(P_s-1)$)
x^*	Spatial variable
s	Dimensionless spatial variable (x^*/w^*)
Δx	Dimensionless spatial increment
Δt	Dimensionless time increment
α,β,γ	Time dependent coefficients appearing in iterative equation (3-1)
ε	Amplitude of disturbance
Λ	Restrictor coefficient

NOMENCLATURE (continued)

SYMBOL	DESCRIPTION
μ	Fluid viscosity
ω_o	Acceleration parameter (of iterative equation (3-1))
Ω^*	Excitation frequency
π	3.141593.......
ρ^*	Density of lubricating gas
σ	Squeeze number $(12\mu\Omega^* w^{*2}/h_o^{*2}p_a^*)$

CHAPTER I

INTRODUCTION

In recent years gas bearing technology has been a key factor in many industrial applications. This trend is simply explained by the fact that gas operating systems have a ready supply of lubricant, and the costly addition of oil pumps, lines, etc., can be eliminated. In addition, gas lubricants offer less friction, are not susceptible to temperature extremes, and exhibit greater reliability.

Even though gas lubrication offers a wealth of advantages, its applicability has been limited mainly by two difficulties: (1) the requirement of close tolerances, (2) the stability problem associated with a compressible fluid. The latter problem results from a load variation which could be attributed to an unbalanced rotating machine or to a high speed operational mode. If the conditions are favorable, dynamic loading can cause self-excited or undamped vibrations [1]*. This behavior is not desirable since it downgrades bearing performance and ultimately causes direct contact between surfaces, leading to the bearing collapse.

To study this problem the lubricating film can be modeled as a damped single-degree of freedom, mass-spring system where the exciting force is supplied by the changing

---

*Numbers in brackets refer to REFERENCES.

load. Usually the load changes are of periodic nature exhibiting components in phase and ninety degrees out of phase with the displacement. Thus, stiffness and damping properties can be attributed to the film. To determine the effect of the exciting force on the film, the Reynolds' equation is solved for the lubricant film pressure distribution. Since the Reynolds' equation (pressure equation) is a nonlinear, time-dependent, parabolic, partial differential equation, it is not surprising that no known exact solutions exist for this equation. Thus, it has been necessary to attack the Reynolds' equation with approximating techniques, or avoid the equation altogether by using lumped parameter methods. The more accurate approximating techniques include small-perturbation theory, "linearized Ph" techniques, and weighted residual methods.

Richardson [2] was one of the first to study the dynamic behavior of an orifice compensated gas bearing using lumped parameter methods. In his analysis load and flow relationships are developed which could be used for design purposes. Licht and Elrod [3] presented an analysis of an orifice gas bearing where the equations of motion are linearized and the stability criteria are based on small deviations from the steady-state operating point. For a similar bearing, they compared their results to the ones obtained by Richardson and observed marked differences between the limiting values of parameters which influence the stability of the bearing.

Mullan and Richardson [4] studied the effect of small eccentricity ratios on the dynamic behavior of an inherently compensated gas journal bearing. They made a parallel analysis using lumped parameter and small-perturbation methods. The bearing coefficients (stiffness and damping) obtained from both theories were compared to experimental data, and it was observed that the experimental results agree reasonably well with both analyses. Lund [5] analyzed a hydrostatic gas journal bearing with journal rotation and vibration. In each case a first-order perturbation solution is developed where small eccentricity ratios and small vibration amplitude is assumed, respectively. Numerical results were given for the load carrying capacity as a function of the bearing number and constant pressure ratios.

Wernick and Pan [6] applied the Reynolds' equation to a self-acting, partial-arc gas journal bearing, and the equation was perturbed in terms of the compressibility number. The resulting equations were solved using the nonlinear Galerkin's technique. The bearing load and stability derivatives were expressed in a power series of the compressibility number, and the presented numerical results retained powers up to and including two. This analysis appears to be promising since it is valid for high eccentricity ratios. Ausman [7] investigated the effect of sinusoidally time-varying loads on the behavior of a gas journal bearing. In approximating the lubricant pressure forces the "linearized Ph" technique was utilized to solve the Reynolds' equation in conjunction

with the equations of journal motion. In addition to the results obtained on the resonance frequency and bearing response it was concluded that nonlinear response terms can be neglected if the total maximum eccentricity is less than one half the average radial clearance.

Stiffler [8] presented an analysis of an inherently compensated, multiple inlet, circular thrust bearing subjected to a periodic load disturbance. The Reynolds' equation was solved by assuming small perturbations of the gas film pressures. The bearing stiffness and damping were obtained by modeling the gas film as a linear spring-dashpot system. Stiffler and Smith [9] carried out a theoretical analysis of the dynamic characteristics of an inherently compensated, square geometry thrust bearing based on small perturbations of the Reynolds' equation. The resulting equations were solved numerically by finite differencing. The bearing stiffness and damping were obtained in a similar way as done by Stiffler for a circular thrust bearing.

Compressible squeeze film theory for large squeeze numbers is of great importance in understanding the Reynolds' equation behavior when it is applied to externally pressurized gas bearings. It has been demonstrated by the works of Salbu [10] and Pan [11] that two parallel surfaces transversely oscillating at high frequencies will develop a mean film pressure above the ambient which could support a load. Thus, the surfaces and gas film as a system operate as a thrust bearing. The work of Sadd and Stiffler [12] on

squeeze film dampers at low squeeze numbers could also be used to predict the behavior of externally pressurized gas bearings under certain conditions. They analyzed the effect of a periodic load disturbance amplitude on the dynamic performance of a squeeze film damper.

Recently, numerical techniques have been used extensively in solving lubrication technology related problems. This can be attributed to the development of fast, accurate and reliable computational algorithms with the help of high speed computing machines [13].

1.1 Statement of the Problem

The analysis of the effect of dynamic loading on an externally pressurized gas bearing is greatly restricted by the availability of solutions to the Reynolds' equation subject to the boundary conditions of a particular problem. The literature survey shows that only small amplitude motion effect on bearing performance has been avoided to date; although, it is very important to know how the bearing design parameters are affected.

The purpose of this report is to carry out a theoretical analysis of the effects of high amplitude oscillation induced by a periodic load disturbance on the dynamic behavior of an inherently compensated, infinite long strip gas thrust bearing. The gas film pressure distribution is described by applying the Reynolds' equation to the strip

shown in Figure 1, with the bearing dimensions normalized by the bearing width.

The Reynolds' equation, a parabolic partial differential equation, is numerically approximated using finite-difference techniques. Thus, the film pressure distribution is numerically described as a function of space and time. The time dependent load carrying capacity is represented by a Fourier series up to and including the third harmonics. Design curves for the stiffness, damping, and load capacity are presented as functions of inlet location, restrictor coefficient, supply pressure, amplitude of oscillation and squeeze number. The theoretical analysis of the bearing characteristics is developed in the following chapter.

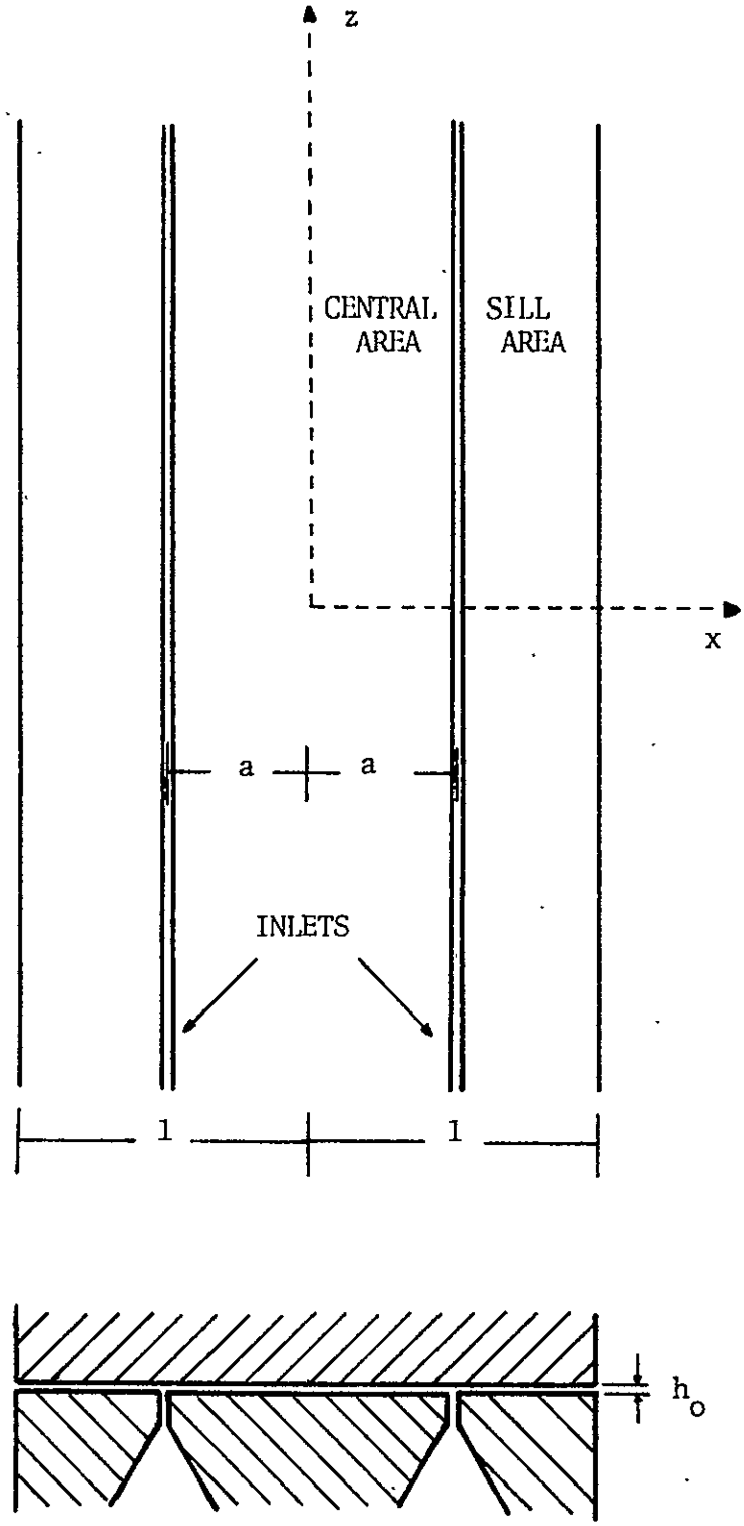

FIGURE 1. Inherently Compensated Strip Thrust Bearing

CHAPTER II

THEORETICAL ANALYSIS

2.1 Mathematical Model

To facilitate the theoretical analysis of the problem, the bearing is divided into three regions.....the inlet region, the central area or region I, and the sill area or region II. When the bearing is in operation, the lubricating gas at a pressure, P_s, is supplied through the inlets. As the gas flows across the sill its pressure changes gradually to ambient state when it reaches the outer edges of the sill.

For simplicity only one-half of the geometry is considered since the bearing is symmetric about the centerline. In addition, the bearing is assumed to be infinitely long in the z-direction, and the inlets are assumed to be an inherent continuous line source where the discharge area is defined by the bearing length and the gas film height. Inlets using discrete holes may be approximated by a line source.

2.1.1 Reynolds' Equation

For compressible fluids and one dimensional geometries the pressure distribution is described by the reduced Reynolds' equation [14]:

$$\frac{\partial}{\partial x^*}\left(h^{*3}\rho^*\frac{\partial P^*}{\partial x^*}\right) = 12\mu\frac{\partial}{\partial t^*}\left(\rho^* h^*\right) \qquad (2\text{-}1)$$

where the starred variables denote actual or real variables as opposed to normalized variables.

The bearing surfaces are considered to be parallel at all times since they do not pivot about any point in the bearing. Thus, the film thickness, h^*, is uniform throughout the bearing. The variation of the thickness in a direction perpendicular to the surfaces is caused by the load disturbance.

To further simplify the problem it is assumed that the gas film behaves as an ideal gas with constant specific heats. Also, the flow is considered isothermal [15] with P^*/ρ^* = constant.

In solving the pressure equation (2-1) the following dimensionless variables are introduced:

$$x = x^*/w^*$$

$$t = t^*\Omega^*$$

$$P = P^*/P_a^*$$

$$h = h^*/h_o^*$$

Thus, the nondimensional Reynolds' equation can be written as follows:

$$\frac{\partial^2}{\partial x^2}(P^2) = \frac{2\sigma}{h^3}\frac{\partial}{\partial t}(Ph) \qquad (2\text{-}2)$$

where the squeeze number, σ, is given by

$$\sigma = \frac{12\mu\Omega^* w^{*2}}{h_o^{*2} P_a^*} \tag{2-3}$$

The film height is a function of time since it depends on the load disturbance. If the load disturbance is of a periodic nature, the lubricant thickness can be modeled as

$$h(t) = 1 + \varepsilon \sin(t) \tag{2-4}$$

where εh_o^* is the amplitude of the displacement about the average film thickness, h_o^*. Substituting equation (2-4) into equation (2-2), the resulting equation can be solved for the pressure distribution in regions I and II subject to the boundary conditions.

2.1.2 Boundary Conditions

The pressure at the edges of the sill is at ambient conditions. Due to the bearing symmetry about the centerline the pressure changes with respect to the x-direction are equal to zero along this line. Thus, the boundary conditions in dimensionless form are

$$P(1,t) = 1 \tag{2-5a}$$

$$\frac{\partial P(0,t)}{\partial x} = 0 \tag{2-5b}$$

In order to specify the pressure at the inlets the dynamic mass flow for each of the three bearing regions is equated

at this point to comply with continuity requirements. The mass flow expressions are derived in section 2.2.1.

In view of the difficulty in obtaining a closed form solution, it was decided to approximate the partial differential equation (2-2) and boundary conditions using finite-difference expressions. In this respect the following static solution of equation (2-2) will be utilized later:

$$P(x,0) = P_o \qquad 0 \leq x \leq a \qquad (2\text{-}6a)$$

$$P^2(x,0) = 1 + \frac{(P_o^2 - 1)}{(1 - a)}(1-x) \qquad a < x \leq 1 \qquad (2\text{-}6b)$$

where, a, is the nondimensional distance between the centerline and the inlets.

2.2 Design Parameters

To appreciate the effects that large amplitude have on the bearing design, it is necessary to investigate how the mass flow, load capacity, stiffness and damping depend on variables such as the displacement amplitude, ε, the squeeze number, σ, the supply pressure, P_s, the characteristics of the restrictors, and the distance between the inlets and the centerline. Expressions for each of these design parameters are derived in the following sections.

2.2.1 Mass Flow

In order to specify the pressure at the inlets continuity requirements must be met; then

$$M_R^* + M_I^* = M_{II}^*$$

where

M_R^* = mass flow through the restrictors,

M_I^* = mass flow from the central area, and

M_{II}^* = mass flow through the sill region.

The total mass flow through two parallel, infinitely long strip surfaces is given by Constantinescu [14] as

$$M_x^* = -\int_{-\infty}^{\infty} \rho^* \frac{h^{*3}}{12\mu} \frac{\partial P^*}{\partial x^*} dz \tag{2-7}$$

It is assumed that the gas is supplied through two line sources each located at a nondimensional distance, a, from the centerline and that the gas flows isothermally. Using equation (2-4) and replacing the density of the equation of state for an ideal gas, the nondimensional form of equation (2-7) becomes

$$M_x = \frac{M_x^*}{m_o^*} = -(1 + \varepsilon \sin(t))^3 \frac{\partial (P^2)}{\partial x} \tag{2-8}$$

where

$$m_o^* = \frac{L^* h_o^{*3} P_a^{*2}}{24\mu (RT) w^*} \tag{2-9}$$

The dimensionless average mass flow at h equal to one is obtained from equation (2-8) by making use of the static pressure distribution, equation (2-6b): therefore,

$$M_o = -\frac{(P_o^2 - 1)}{(1 - a)} \tag{2-10}$$

Applying equation (2-8) to region I and evaluating it at $x = a$ (inlets),

$$M_I = -(1 + \varepsilon \sin(t))^3 \left. \frac{\partial (P^2)}{\partial x} \right|_{x = a^-} \tag{2-11}$$

Likewise for region II,

$$M_{II} = -(1 + \varepsilon \sin(t))^3 \left. \frac{\partial (P^2)}{\partial x} \right|_{x = a^+} \tag{2-12}$$

The total dimensionless mass flow through the restrictors [16] is given by

$$M_R = \frac{M_R{}^*}{m_{R1}{}^*} = (1 + \varepsilon \sin(t)) \left[\frac{P}{P_s}\right]^{1/k} \left[1 - \left[\frac{P}{P_s}\right]^{\frac{k-1}{k}}\right]^{1/2} \tag{2-13}$$

where

$$m_{R1}{}^* = \frac{C_D\ L^*\ h_o{}^*\ P_a{}^*\ P_s}{\sqrt{RT}} \left[\frac{2g_o k}{k-1}\right]^{1/2} \tag{2-14}$$

and

$$\frac{P}{P_s} > \left[\frac{2}{k+1}\right]^{\frac{k}{k-1}}$$

The corresponding dimensionless average mass flow is

$$M_{RO} = \left[\frac{P_o}{P_s}\right]^{1/k} \left[1 - \left[\frac{P_o}{P_s}\right]^{\frac{k-1}{k}}\right]^{1/2} \tag{2-15}$$

When the flow is critical,

$$M_R = \frac{M_R{}^*}{m_{R2}{}^*} = (1 + \varepsilon \sin(t)) \qquad (2\text{-}16)$$

where

$$m_{R2}{}^* = \frac{C_D \; L^* \; h_o{}^* \; P_a{}^* \; P_s}{\sqrt{RT}} \left[\frac{2g_o k}{k+1}\right]^{1/2} \left[\frac{2}{k+1}\right]^{\frac{1}{k-1}} \qquad (2\text{-}17)$$

and

$$\frac{P}{P_s} \leq \left[\frac{2}{k+1}\right]^{\frac{k}{k-1}}$$

with the average mass flow given by

$$M_{RO} = 1 \qquad (2\text{-}18)$$

Equating the average mass flow through the restrictors and the sill, the pressure P_o downstream of the inlets can be obtained implicitly from

$$\frac{\Lambda \; P_s{}^2}{(P_o{}^2-1)} \left[\frac{P_o}{P_s}\right]^{1/k} \left[1 - \left[\frac{P_o}{P_s}\right]^{\frac{k-1}{k}}\right]^{1/2} = 1 \qquad (2\text{-}19)$$

when

$$\frac{P_o}{P_s} > \left[\frac{2}{k+1}\right]^{\frac{k}{k-1}}$$

or explicitly from

$$P_o^2 = 1 + \Lambda P_s^2 \left[\frac{2}{k+1}\right]^{\frac{1}{k-1}} \left[\frac{k-1}{k+1}\right]^{1/2} \tag{2-20}$$

when

$$\frac{P_o}{P_s} \leq \left[\frac{2}{k+1}\right]^{\frac{k}{k-1}}$$

where the restrictor coefficient, Λ , is

$$\Lambda = \frac{24C_D \ \mu \ w^* \ (1-a)}{h_o{}^{*2} \ P_a{}^* \ P_s} \left[\frac{2g_o kRT}{k-1}\right]^{1/2} \tag{2-21}$$

The restrictor coefficient, Λ , is a dimensionless number which groups the bearing dimensions and gas properties. Physically the restrictor coefficient represents the ratio of the mass flow resistance across the sill area to the mass flow resistance through the inlets.

To calculate the static pressure, P_o, downstream of the inlets, equations (2-19) and (2-20) are used for subcritical and critical flow, respectively. Knowing the pressure P_o the static pressure distribution (initial conditions), equations (2-6a) and (2-6b), can be obtained.

Since the sum of the flow through the inlets and central region must equal the flow across the sill, the dynamic pressure downstream of the inlets at x = a can be found from the following equation:

$$\left.\frac{\partial (P^2)}{\partial x}\right|_{x=a^-} - \left.\frac{\partial (P^2)}{\partial x}\right|_{x=a^+} = \frac{\Lambda P_s^{\,2}}{(1+\varepsilon\sin(t))^2(1-a)}\left[\frac{P}{P_s}\right]^{1/k}\left[1-\left(\frac{P}{P_s}\right)^{\frac{k-1}{k}}\right]^{1/2}$$

(2-22)

when

$$\frac{P}{P_s} > \left[\frac{2}{k+1}\right]^{\frac{k}{k-1}}$$

or

$$\left.\frac{\partial (P^2)}{\partial x}\right|_{x=a^-} - \left.\frac{\partial (P^2)}{\partial x}\right|_{x=a^+} = \frac{\Lambda P_s^{\,2}}{(1+\varepsilon\sin(t))^2(1-a)}\left[\frac{k-1}{k+1}\right]^{1/2}\left[\frac{2}{k+1}\right]^{\frac{1}{k-1}}$$

(2-23)

when

$$\frac{P}{P_s} \leq \left[\frac{2}{k+1}\right]^{\frac{k}{k-1}}$$

If the displacement amplitude, ε, and/or the squeeze number, σ, are high enough, the mass flow could reverse through the inlets. The pressure downstream of the orifices becomes the supply pressure and the supply pressure, P_s, acts as backpressure. Then, equations (2-22) and (2-23) are modified as follows:

$$\left.\frac{\partial (P^2)}{\partial x}\right|_{x=a^+} - \left.\frac{\partial (P^2)}{\partial x}\right|_{x=a^-} = \frac{\Lambda P_s\, P}{(1+\varepsilon\sin(t))^2(1-a)}\left[\frac{P_s}{P}\right]^{1/k}\left[1-\left(\frac{P_s}{P}\right)^{\frac{k-1}{k}}\right]^{1/2}$$

(2-24)

when

$$\frac{P_s}{P} > \left[\frac{2}{k+1}\right]^{\frac{k}{k-1}}$$

or

$$\left.\frac{\partial (P^2)}{\partial x}\right|_{x=a^+} - \left.\frac{\partial (P^2)}{\partial x}\right|_{x=a^-} = \frac{\Lambda P_s P}{(1+\varepsilon\sin(t))^2(1-a)}\left[\frac{k-1}{k+1}\right]^{1/2}\left[\frac{2}{k+1}\right]^{\frac{1}{k-1}} \tag{2-25}$$

when

$$\frac{P_s}{P} \leq \left[\frac{2}{k+1}\right]^{\frac{k}{k-1}}$$

2.2.2 Load Capacity

The time dependent load capacity is given by

$$W(t) = \frac{W^*}{2w^* L^* P_a^*} = \int_0^1 [P(x,t) - 1]\, dx \tag{2-26}$$

The pressure $P(x,t)$ can be obtained from the Reynolds' equation (2-2). Additionally the restrictor coefficient must be given in order to specify the pressure at the inlets using any one of the equations (2-22),(2-23),(2-24) or (2-25) according to the flow intensity and direction.

The load capacity as a function of time can be expressed in the form of the Fourier series:

$$W(t) = A_o + A_1\cos(t) + B_1\sin(t) + A_2\cos(2t) + B_2\sin(2t) + \ldots\ldots \tag{2-27}$$

Using the orthogonality properties the coefficients are given by

$$A_o = \overline{W} = \frac{1}{2\pi} \int_0^{2\pi} W(t)dt \tag{2-28a}$$

$$A_n = \frac{1}{\pi} \int_0^{2\pi} W(t)\cos(nt)dt \tag{2-28b}$$

$$B_n = \frac{1}{\pi} \int_0^{2\pi} W(t)\sin(nt)dt \tag{2-28c}$$

Since the film pressure distribution is calculated by numerical means, the time dependent load capacity is determined at discrete points; therefore, equations (2-27), (2-28a), (2-28b) and (2-28c) must also be computed by numerical means.

2.2.3 Stiffness and Damping

The bearing motion is linearly approximated by

$$W^*(t) = - K^* Y^* - D^* \dot{Y}^* + \overline{W} \tag{2-29}$$

where

$$Y^* = \varepsilon h_o^* \sin(\Omega^* t^*)$$

and the symbols K^* and D^* represent the linear stiffness and damping, respectively. If equation (2-29) is divided through by the ambient pressure and the bearing area, it can be written as

$$W(t) = \frac{W^*}{2w^*\ L^*P_a^*} = -\frac{K^*\varepsilon h_o^*}{2w^*\ L^*P_a^*}\sin(t) - \frac{D^*\Omega^*\varepsilon h_o^*}{2w^*\ L^*P_a^*}\cos(t) \tag{2-30}$$

By matching the foregoing equation with equation (2-27) and introducing the squeeze number, σ, a dimensionless stiffness and damping are defined by

$$K_s = \frac{B_1}{(P_s-1)\varepsilon} = \frac{K^*\ h_o^*}{2w^*\ L^*\ P_a^*(P_s-1)} \tag{2-31}$$

$$D = \frac{12A_1}{\varepsilon\sigma}(1-\varepsilon^2)^{3/2} = \frac{D^*\ (1-\varepsilon^2)^{3/2}}{2\mu L^*(w^*/h_o^*)^3} \tag{2-32}$$

CHAPTER III

NUMERICAL ANALYSIS

This chapter presents the finite difference formulations of equation (2-2) and boundary conditions, and the solution technique of the load coefficient integrals. In addition, a summary of the computation procedure is presented, and a brief explanation of the calculation's global error is given.

3.1 Reynolds' Equation

The Reynolds' equation in its nondimensional form, equation (2-2), is approximated with second-order finite difference expressions. The space and time derivatives are represented with central and backward difference expressions, respectively. Since a three-point backward difference expression is used to represent the time derivative, the resulting finite difference equation is of the implicit type. The use of an implicit formulation assures unconditional stability regardless of the time and space increment size. The point S.O.R. (successive-over-relaxation) method is conveniently utilized to converge the implicit formulation*, and it is written as follows:

---

*The finite difference expressions and the derivation of equation (3-1) are given in Appendix A.

$$P_i^{\#} = \frac{1}{3+2\alpha P_i^{(s)}} \alpha \left[P_{i+1}^{2\,(s)} + P_{i-1}^{2\,(s+1)}\right] + 4\beta P_{i,n-1} - \gamma P_{i,n-2} \tag{3-1}$$

and the computation is accelerated using the following equation

$$P_i^{(s+1)} = \omega_o P_i^{\#} + (1-\omega_o) P_i^{(s)} \tag{3-2}$$

where i and n are the spatial and time node indexes, and (s) is an iteration counter. The variable ω_o is the acceleration parameter for the foregoing point S.O.R. formulation; also, the coefficients α, β, and γ are given below:

$$\alpha = \frac{h_{i,n}^2 \, \Delta t}{\sigma \, \Delta x^2} = \frac{(1 + \varepsilon \sin(t))^2 \Delta t}{\sigma \Delta x^2}$$

$$\beta = \frac{h_{i,n-1}}{h_{i,n}} = \frac{(1 + \varepsilon \sin(t-\Delta t))}{(1 + \varepsilon \sin(t))}$$

$$\gamma = \frac{h_{i,n-2}}{h_{i,n}} = \frac{(1 + \varepsilon \sin(t-2\Delta t))}{(1 + \varepsilon \sin(t))} \tag{3-3}$$

It is desirable to iterate equation (3-1) using the optimum acceleration parameter in equation (3-2) so the highest rate of convergence possible is attained, but unfortunately there is no theory or a set procedure on the calculation of this optimum value for nonlinear formulations.

Qualitative information on the optimum value or neighboring parameters can be inferred from the theory of S.O.R. applied to linear cases as suggested by Mitchell [17] and Roache [18]. In addition, a systematic numerical search is implemented in order to establish a small region where the true optimum acceleration parameter lies.

3.2 Boundary Conditions

The boundary conditions (2-5a) and (2-5b)* are incorporated into the difference equation, obtained in section 3.1, as follows:

$$P_{i,n} = \frac{4P_{i+1,n} - P_{i+2,n}}{3} \quad , \; i = 1 \qquad \text{(3-4a)}$$

$$P_{i,n} = 1 \qquad , \; i = MAX \qquad \text{(3-4b)}$$

where i = 1 coincides with the centerline, and i = MAX the maximum number of spatial nodes, indicates the outer edges of the sill area. The pressure at the inlets given by any one of the equations (2-22), (2-23), (2-24), and (2-25) is approximated with finite differences** yielding

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* Difference formula is given in Appendix A.

** Derivation of finite difference equations is given in Appendix A.

$$P_{i,n}^{(r+1)} = \frac{2}{3}\left(P_{i+1,n}^{2} + P_{i-1,n}^{2}\right) - \frac{1}{6}\left(P_{i+2,n}^{2} + P_{i-2,n}^{2}\right)$$

$$+ \frac{\Delta x\ \Lambda\ P_s^{2}}{3(1+\varepsilon\sin(t))^{2}(1-a)} \left[\frac{P_{i,n}^{(r)}}{P_s}\right]^{1/k} \left[1-\left[\frac{P_{i,n}^{(r)}}{P_s}\right]^{\frac{k-1}{k}}\right]^{1/2} {}^{1/2} \tag{3-5}$$

when

$$P_{i,n} < P_s\ ,\ \frac{P_{i,n}}{P_s} > \left[\frac{2}{k+1}\right]^{\frac{k}{k-1}}$$

or

$$P_{i,n} = \frac{2}{3}\left(P_{i+1,n}^{2} + P_{i-1,n}^{2}\right) - \frac{1}{6}\left(P_{i+2,n}^{2} + P_{i-2,n}^{2}\right)$$

$$+ \frac{\Delta x\ \Lambda\ P_s^{2}}{3(1+\varepsilon\sin(t))^{2}(1-a)} \left[\frac{k-1}{k+1}\right]^{1/2} \left[\frac{2}{k+1}\right]^{\frac{1}{k-1}} \tag{3-6}$$

when

$$P_{i,n} < P_s\ ,\ \frac{P_{i,n}}{P_s} \leq \left[\frac{2}{k+1}\right]^{\frac{k}{k-1}}$$

or

$$P_{i,n}^{(r+1)} = \left\{ \frac{2}{3} (P_{i+1,n}^{\ 2} + P_{i-1,n}^{\ 2}) - \frac{1}{6} (P_{i+2,n}^{\ 2} + P_{i-2,n}^{\ 2}) - \frac{\Delta x \ \Lambda \ P_s^{\ 2}}{3(1+\varepsilon\sin(t))^2(1-a)} \left[\frac{P_{i,n}^{(r)}}{P_s}\right]^{\frac{k-1}{k}} \left[1-\left[\frac{P_s}{P_{i,n}^{(r+1)}}\right]^{\frac{k-1}{k}}\right]^{1/2} \right\}^{1/2} \quad (3\text{-}7)$$

when

$$P_{i,n} > P_s \ , \ \frac{P_s}{P_{i,n}} > \left[\frac{2}{k+1}\right]^{\frac{k}{k-1}}$$

or

$$P_{i,n} = \frac{1}{12}\left\{\left[\frac{2 \ \Delta x \ \Lambda P_s}{(1+\varepsilon\sin(t))^2(1-a)} \left[\frac{k-1}{k+1}\right]^{1/2} \left[\frac{2}{k+1}\right]^{\frac{1}{k-1}}\right]^2 + 24\left[4(P_{i+1,n}^{\ 2} + P_{i-1,n}^{\ 2}) - (P_{i+2,n}^{\ 2} + P_{i-2,n}^{\ 2})\right]\right\}^{1/2}$$

$$- \frac{\Delta x \ \Lambda \ P_s}{6(1+\varepsilon\sin(t)) \ (1-a)} \left[\frac{k-1}{k+1}\right]^{1/2} \left[\frac{2}{k+1}\right]^{\frac{1}{k-1}} \quad (3\text{-}8)$$

when

$$P_{i,n} > P_s \ , \ \frac{P_s}{P_{i,n}} \leq \left[\frac{2}{k+1}\right]^{\frac{k}{k-1}}$$

where (r) is an iteration counter.

3.3 Integration

The load carrying capacity given by equation (2-25) is numerically approximated by using Simpson's 1/3 Rule as follows

$$W \approx \frac{\Delta x}{3} \sum_{IN=2}^{M} \left[P_{i-1} + 4P_i + P_{i+1}\right] - 1 \tag{3-9}$$

with IN = 2,4,6, . . . M

When equation (3-9) is repeatedly applied at each time level over the period 2π, the resulting set of values represent the load capacity distribution over the period 2π. In order to calculate the load coefficients given by the integrals (2-28a), (2-28b), and (2-28c), it is necessary to use numerical integration since the load capacity distribution, $W(t)$, is discretely described as explained above.

To approximate the load coefficient integrals a Gaussian Quadrature is implemented for each integral. The Gauss' formula [19] for an arbitrary interval is utilized. Thus, the integrals (2-28a), (2-28b), and (2-28c) are calculated using the following algorithms:

$$\overline{W} \approx \frac{1}{2} \sum_{j=1}^{N} w_j \; W(y_j) \tag{3-10}$$

$$A_n \approx \sum_{j=1}^{N} w_j \; W(y_j) \; \cos(nt) \tag{3-11}$$

$$B_n \approx \sum_{j=1}^{N} w_j \, W(y_j) \sin(nt) \tag{3-12}$$

where $y_j = \pi(x_j+1)$

with $j = 1,2,3, \ldots . N$

and where N is the desired number of points. The terms x_j and w_j correspond to the abscissas and weight factors [19] for Gaussian integration, respectively.

Due to the interval irregularity of abscissas and weight factors, the time levels of calculation are not separated by equal size intervals. However, this condition does not interfere with this study since the main goal is the calculation of the load coefficients over the period 2π.

3.4 Computation Procedure

The implicit numerical formulation for the Reynolds' equation consists of a two-part procedure at each time step: a) a new value of the inlet pressure is iteratively calculated using any one of the finite-difference equations (3-5), (3-6), (3-7), or (3-8) according to the flow intensity and direction, and b) with a new inlet pressure value the pressure distribution in regions I and II are separately iterated until convergency using the finite-difference equation (3-1). Parts a) or outer iteration and b) or inner iterations are repeated until the inlet pressure converges to a desired tolerance. Then, the pressure distribution throughout the bearing is said

to be converged for the time step.

To calculate the load coefficients the solution is started using the static pressure distribution given by the equations (2-6a) and (2-6b), and then the above procedure is repeated at each time level up to and including 2π. Simultaneously, the bearing load capacity at each time level is calculated using Simpson's 1/3 Rule, equation (3-9). Once the load capacity distribution over the period 2π is obtained, the sums (3-10), (3-11), and (3-12) are performed to give the load coefficients.

3.5 Error Analysis

The error associated with the finite-difference equation (3-1) is introduced by the truncation error of the difference formulas used to represent the space and time derivatives in equation (2-2). The combined error of the difference equation is of the order $O(\Delta x^2, \Delta t^2)$, since the central and a three-point backward difference expressions are used to represent the space and time derivatives, respectively. The pressure derivatives in the boundary conditions are approximated with one-sided, second-order difference expressions, so the second order accuracy is preserved. The calculation of the load capacity at each time level introduces an error of the order of the product of Δx^5 and the fourth spatial derivative as described by the remainder of the compound Simpson's 1/3 Rule [19]. In the case of the load coefficient integrals, the error

associated with the calculation is given by the remainder of the Gauss' formula [19] which is of the order of the product of a coefficient function of the number of points desired and the $2N^{th}$ derivative of the function to be integrated where N is the number of points desired. If N is large, the coefficient function of the number of points becomes very small.

Since integration is a smoothing process, it is quite reasonable to state that the numerical work done in this study has a global error at least of order $O(\Delta x^2, \Delta t^2)$.

At each time level convergence was assumed to be attained when the absolute value of the maximum change in the field (that is, regions I and II) from one iteration to the next was less than some specified* change and when the absolute value of the change from one iteration to the next of the inlet pressure was less than some specified tolerance.

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* Different tolerances are used for the range of ε and they are listed in Chapter IV.

CHAPTER IV

RESULTS

The results obtained in this study comprise the load coefficient as a function of the restrictor coefficient, supply pressure, restrictor location, amplitude of oscillation and squeeze number. The average load capacity, and the linear stiffness and damping are generated using the coefficients A_o, B_1 and A_1, respectively, as described by the equations (2-28a), (2-31) and (2-32). These results, to be presented in the following sections of this chapter, are valid for the squeeze number, $\sigma = 0.1$; however, they can be extended to a range of small squeeze numbers ($\sigma < 5$) since the linear stiffness and damping are insensitive to changes of the squeeze number in this range. The non-dimensional distance between the inlet and the centerline is $a = 0.5$. In the computation of the load coefficients, it was necessary to provide the convergence tolerances of the various iterative equations listed in Chapter III. These tolerances varied according to the value of the amplitude of oscillation and they are listed as follows:

1. for $\varepsilon = 0.1, 0.3$

   regions I and II ..... 10^{-7}

   inlet pressure ..... 10^{-6}

2. for $\varepsilon = 0.5$

   regions I and II ..... 10^{-6}

inlet pressure ..... 10^{-5}

The reason for using very tight tolerances in the pressure distribution calculation is due to the need of high accuracy in the calculation of the load coefficients which are relatively small in magnitude.

The dimensionless mass flow is defined as

$$M_o = \frac{(P_o^2 - 1)}{(1-a)}$$

where "a" must be given and P_o can be obtained from equations (2-19), or (2-20) accordingly to the mass flow intensity. The total actual mass flow is given by

$$m_o^* = \frac{L^* \, h_o^* \, P_a^{*2}}{12\mu \; RT \; w^*} \; M_o$$

Design curves for the average load capacity, stiffness, and damping are discussed below.

4.1 Load Coefficients

The load coefficients equation (2-28) were computed for a range ε = 0.1, 0.3, 0.5. These coefficients, functions of the restrictor coefficient and supply pressure, are presented in graphical form as represented by the average load capacity, stiffness, and damping and in table form for the first, second, and third harmonics.

4.1.1 Load Capacity

The nondimensional average load capacity, $\overline{W}$, further normalized by (P_s-1) is presented as a function of the restrictor coefficient and supply pressure in Figures 2-4. The normalized average load capacity is defined by

$$\overline{W}_o = \frac{\overline{W}}{(P_s-1)} = \frac{A_o}{(P_s-1)} = \frac{\overline{W}^*}{2w^*\ L^*\ P_a^*(P_s-1)}$$

For each one of the amplitudes the average load capacity increases substantially from low to high restrictor coefficient. There is some curve leveling in the range of very low and very high restrictor coefficient. At a given restrictor coefficient (i.e., a fixed P_o) the only appreciable change of the load capacity with ε is observed in the range of very low restrictor coefficient; however, this change is not significant.

In general the actual average load capacity is a strong function of the restrictor coefficient and increases considerably as the supply pressure increases.

4.1.2 Damping

The dimensionless damping varies considerably for a wide range of restrictor coefficients and supply pressures as depicted in Figures 5-7. For the geometry, a = 0.5 and within the given amplitude range, for supply pressures up to P_s = 10.0 instability (negative damping) does not occur. For high supply pressures maximum damping occurs at $\Lambda \simeq 4$

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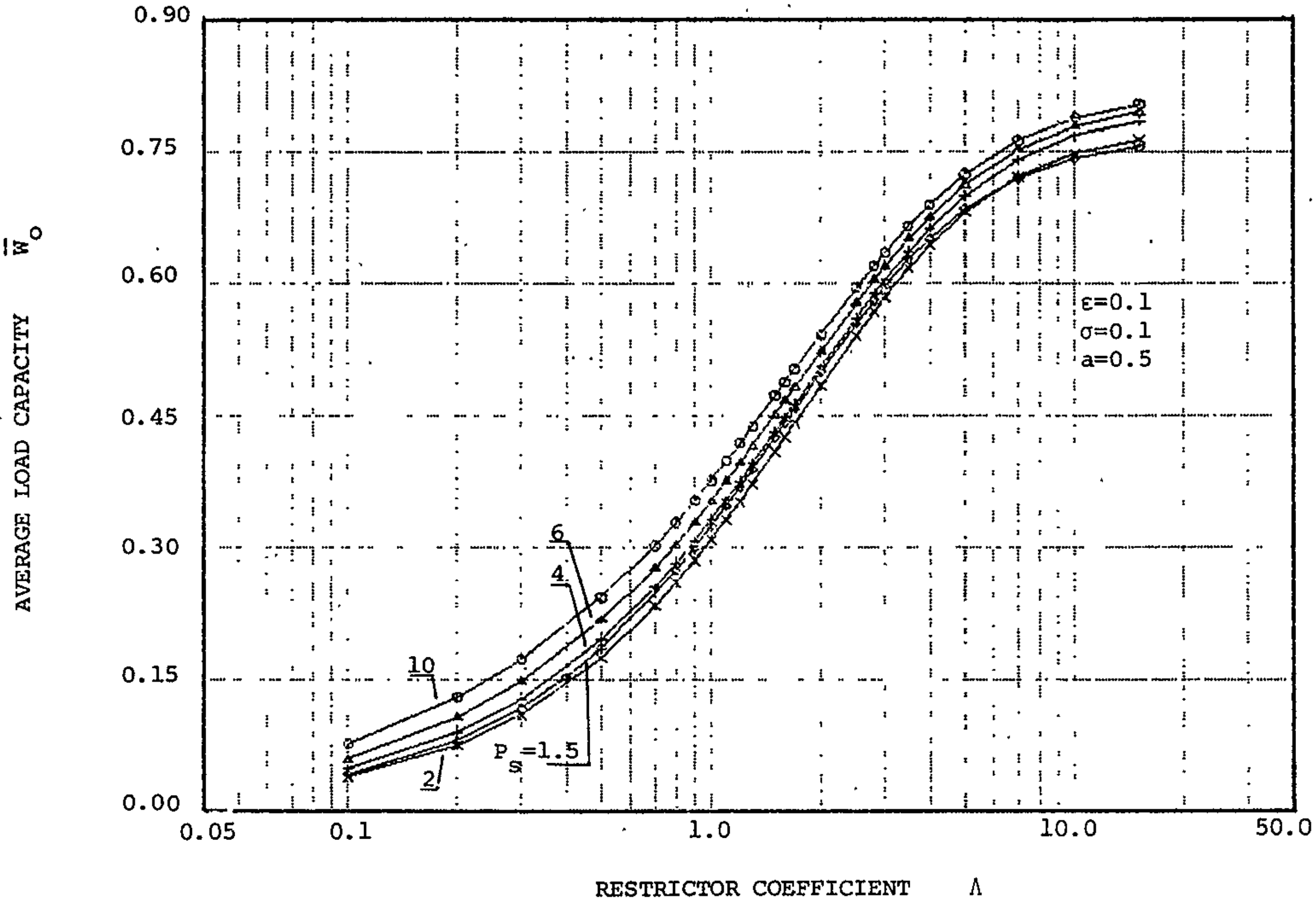


FIGURE 2. Dimensionless Average Load Capacity versus Restrictor Coefficient (ε = 0.1, σ = 0.1, a = 0.5)

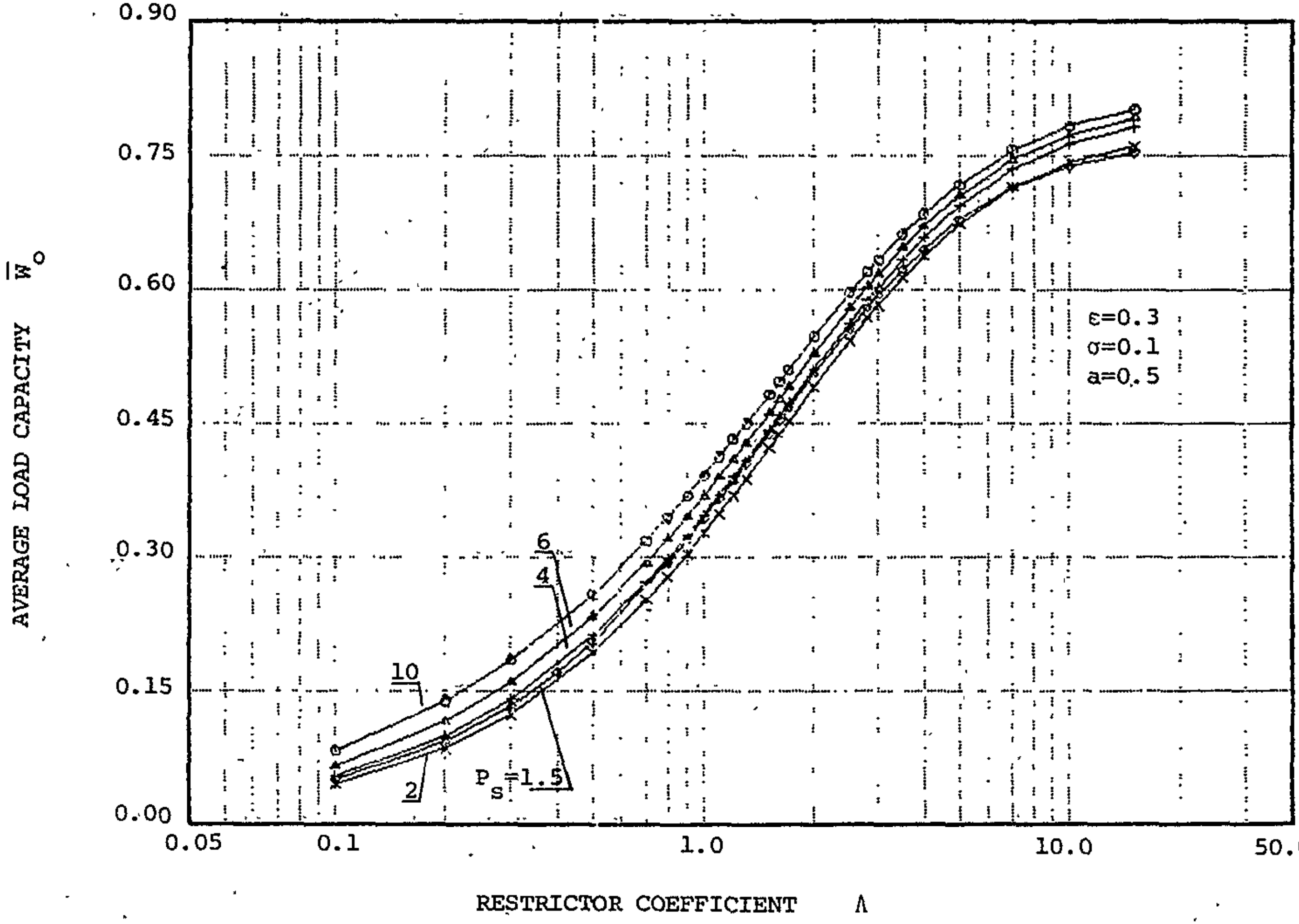


FIGURE 3. Dimensionless Average Load Capacity versus Restrictor Coefficient ($\varepsilon = 0.3$, $\sigma = 0.1$, $a = 0.5$)

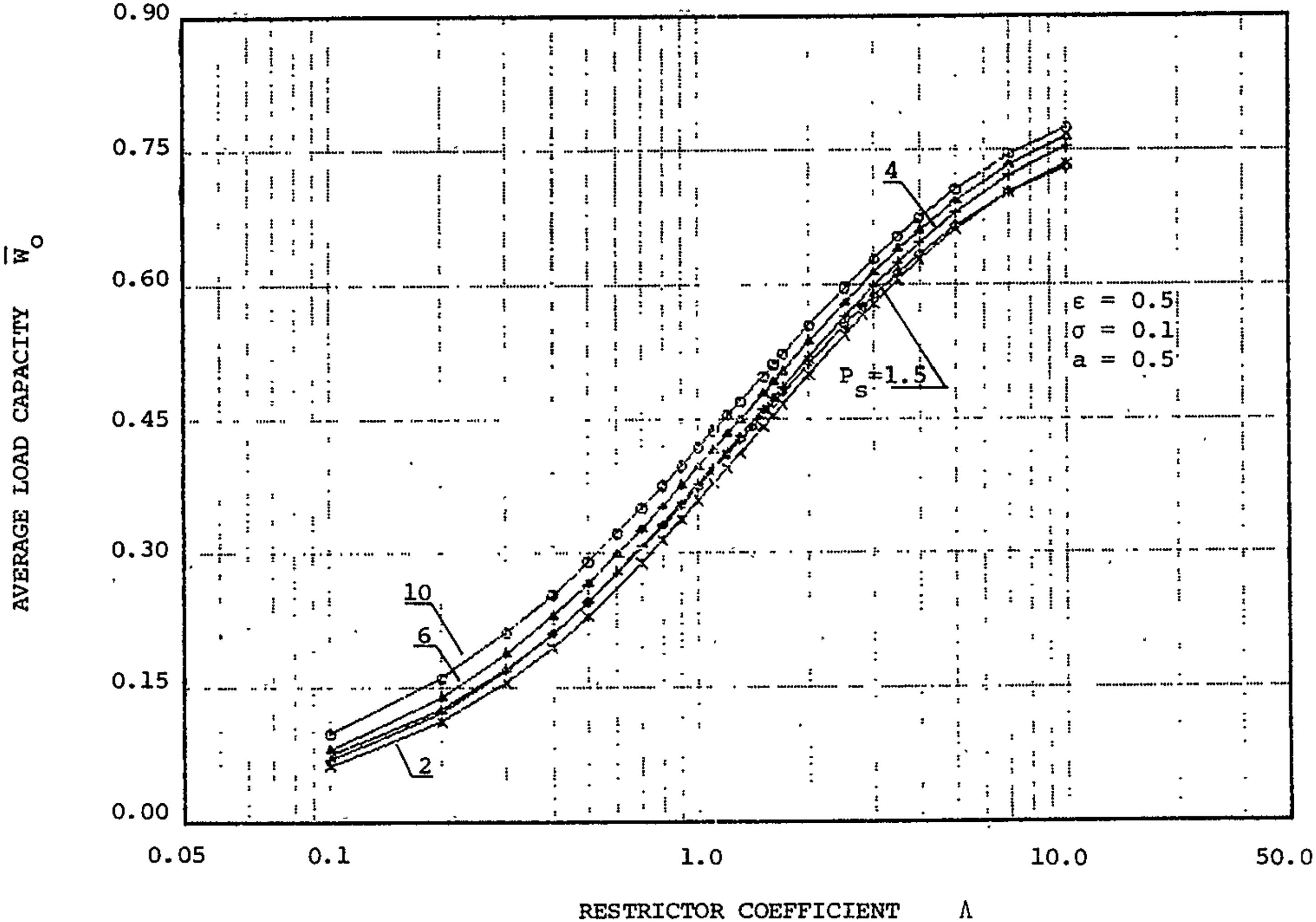


FIGURE 4. Dimensionless Average Load Capacity versus Restrictor Coefficient ($\varepsilon = 0.5$, $\sigma = 0.1$, $a = 0.5$)

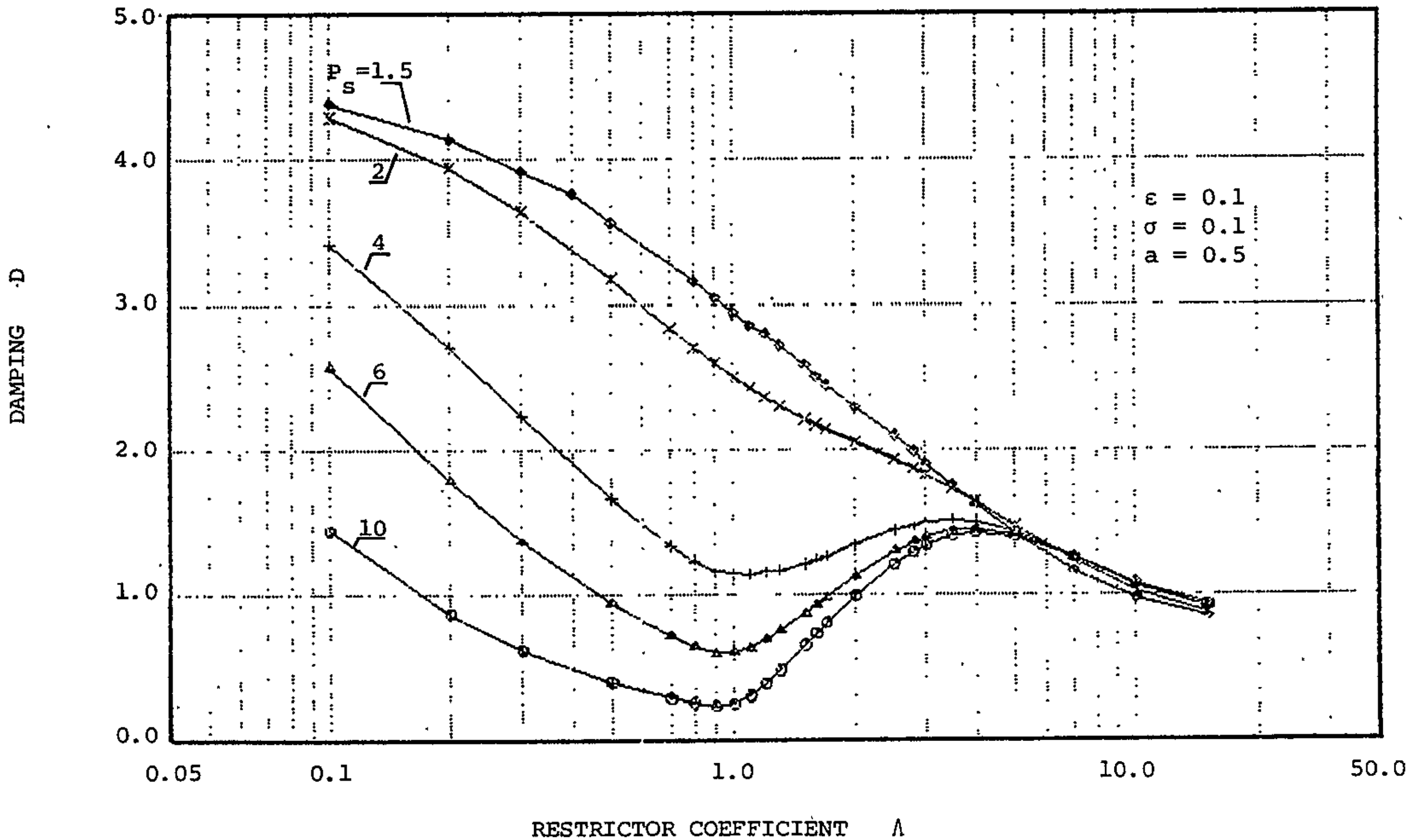


FIGURE 5. Dimensionless Damping versus Restrictor Coefficient ($\varepsilon = 0.1$, $\sigma = 0.1$, $a = 0.5$)

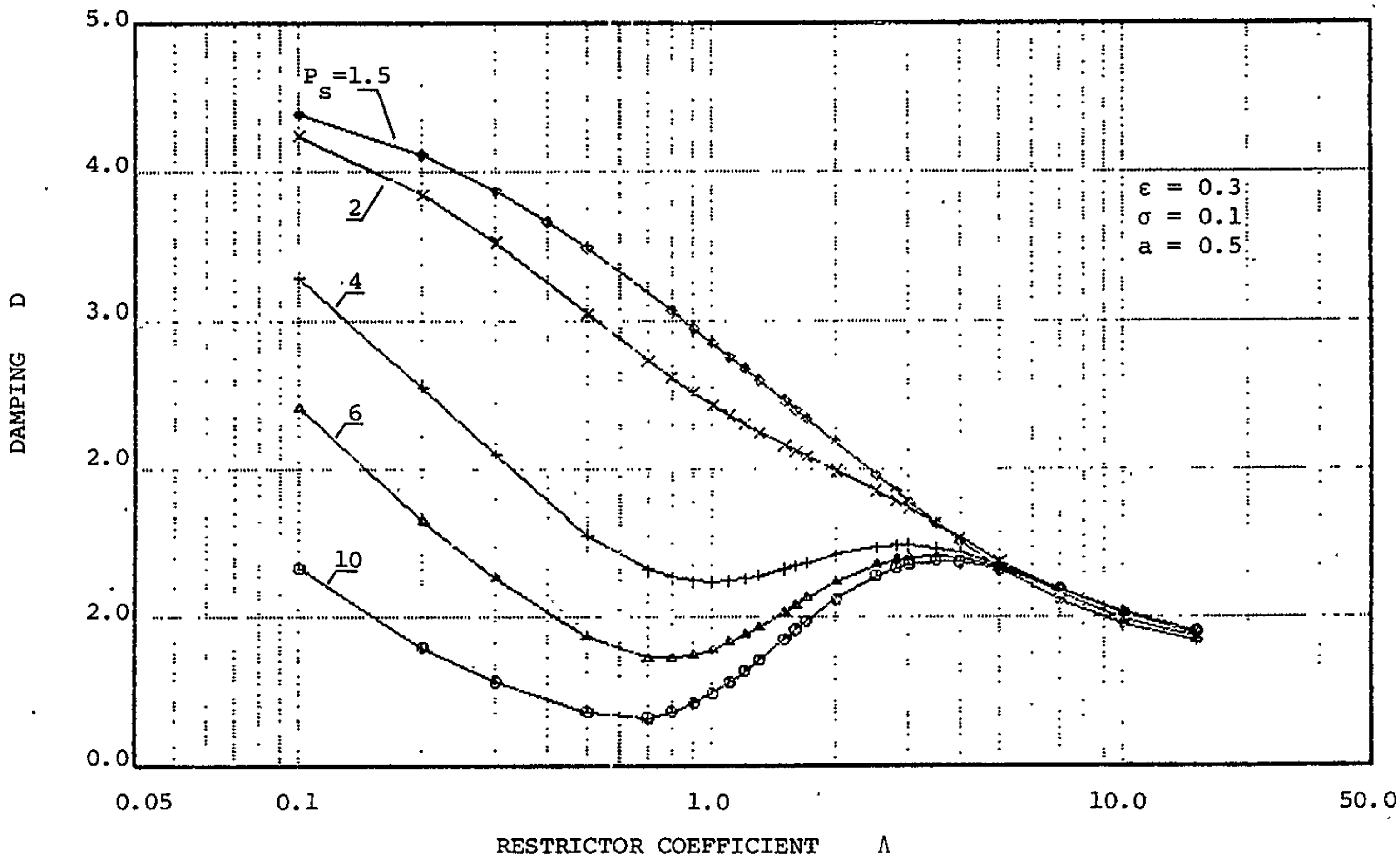


FIGURE 6. Dimensionless Damping versus Restrictor Coefficient ($\varepsilon = 0.3$, $\sigma = 0.1$, $a = 0.5$)

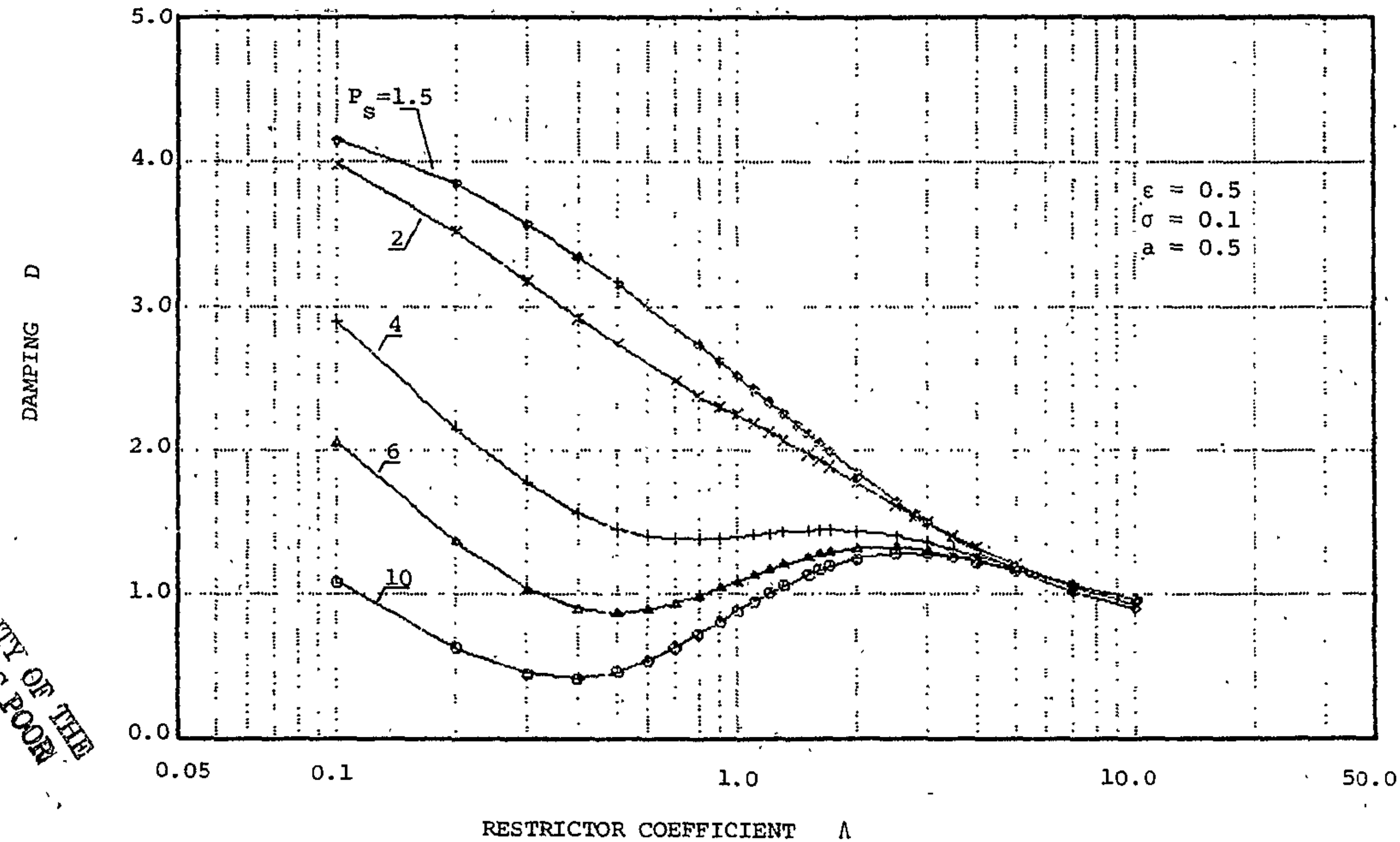


FIGURE 7. Dimensionless Damping versus Restrictor Coefficient (ε = 0.5, σ = 0.1, a = 0.5)

and at very low restrictor coefficient where stiffness is low (see next section). In the case of low supply pressure, damping increases toward a maximum as the restrictor coefficient approaches zero. At high supply pressures the damping goes through a minimum at $\Lambda \approx 1$. As the disturbance amplitude increases this minimum shifts toward lower values of the restrictor coefficient.

The actual damping can be obtained from equation (2-32). It is noteworthy that the factor $(1-\varepsilon^2)^{1.5}$ essentially eliminates major changes in the dimensionless damping due to changes in amplitude ε. This factor was borrowed from Sadd and Stiffler [12], and it was suggested by the fact that the bearing approaches a pure squeeze films at extremes of the restrictor coefficient.

4.1.3 Stiffness

The nondimensional stiffness depends upon the restrictor coefficient and supply pressure as shown in Figures 8-10. It can be observed that the stiffness is highly sensitive to the changes of the restrictor coefficient. As the restrictor coefficient approaches zero, the stiffness becomes very small, whereas the damping approaches a maximum. The stiffness reaches a maximum in the range $0.8 < \Lambda < 2.5$ where some shifting toward $\Lambda \approx 0.8$ occurs as ε approaches 0.5. In the case of very high restrictor coefficients, again the stiffness becomes very small. It is important to note that the dimensionless stiffness is little affected by the amplitude, at least in the range

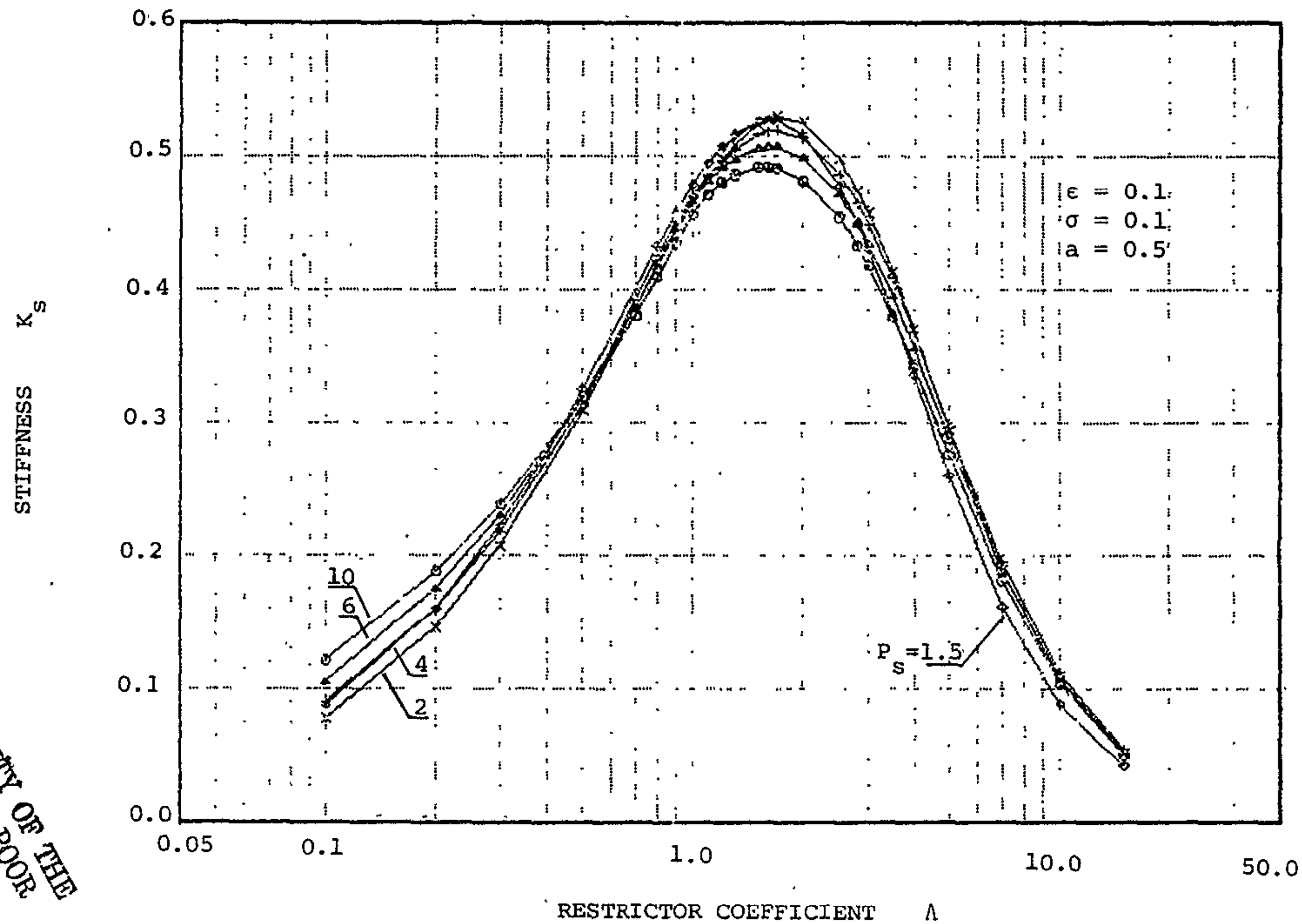


FIGURE 8. Dimensionless Stiffness versus Restrictor Coefficient (ϵ=0.1, σ=0.1, a=0.5)

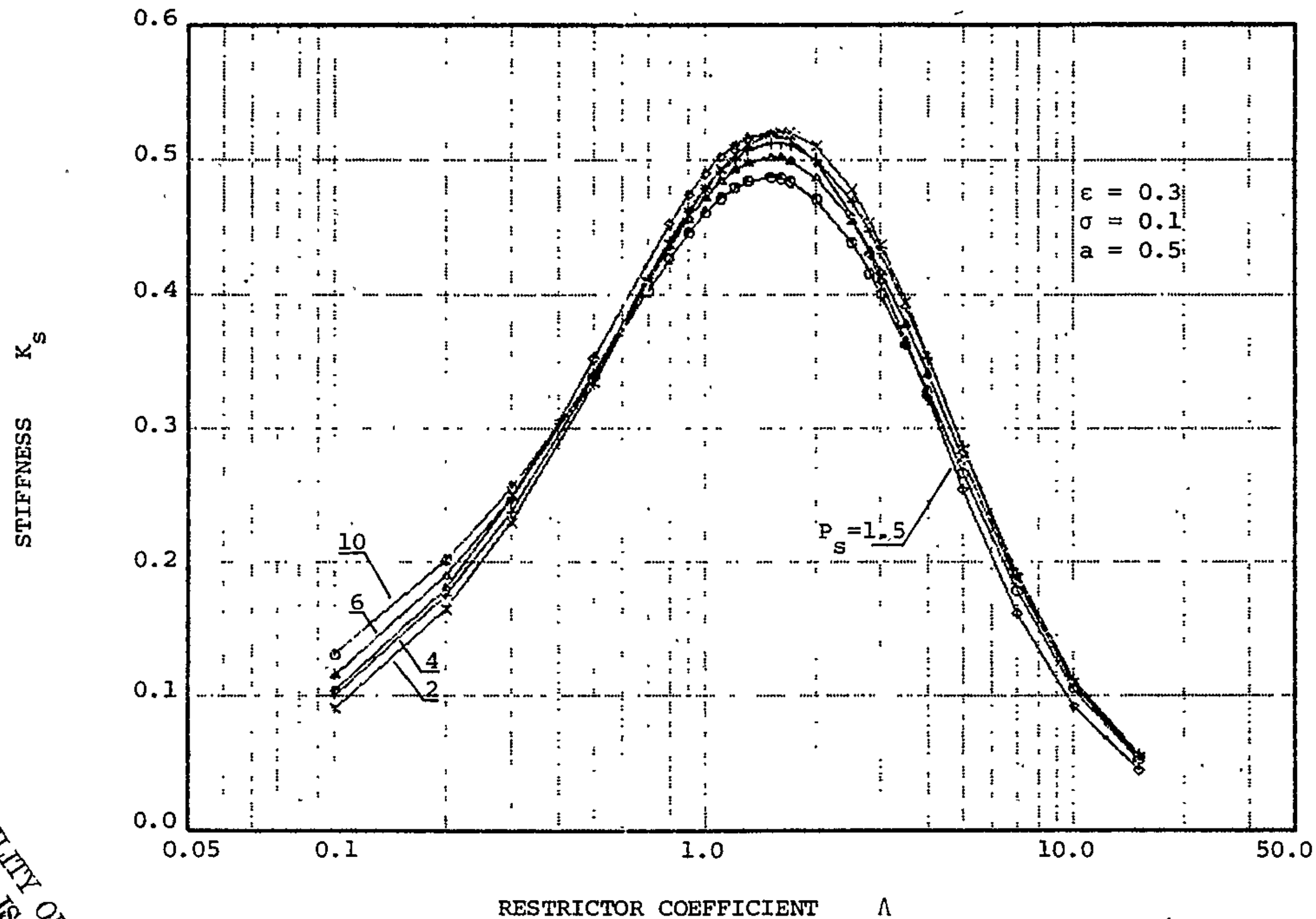


FIGURE 9. Dimensionless Stiffness versus Restrictor Coefficient (ε=0.3,σ=0.1,a=0.5)

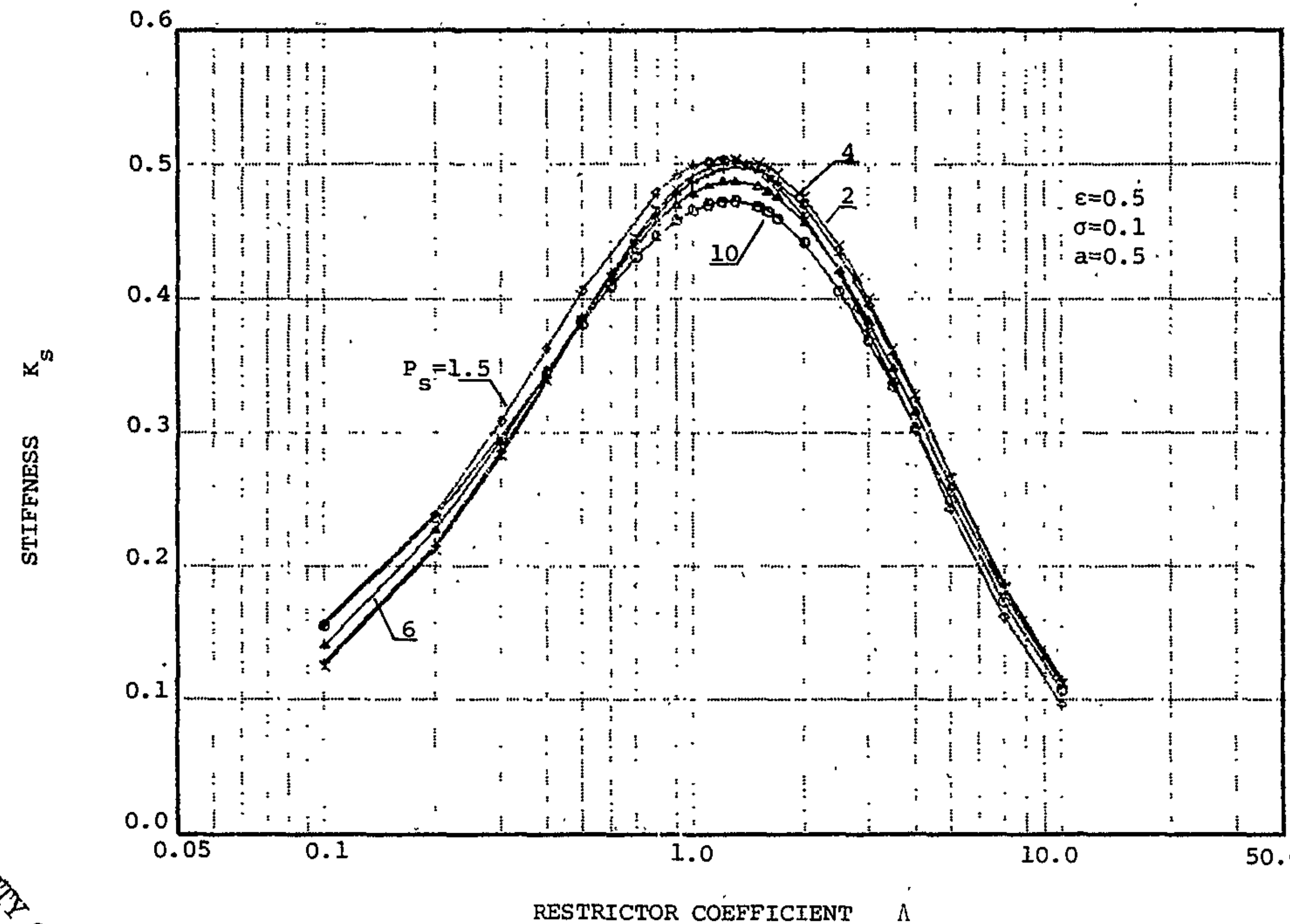


FIGURE 10. Dimensionless Stiffness versus Restrictor Coefficient (ε=0.5,σ=0.1,a=0.5)

investigated.

In the design process the designer should be aware of the following three points:

(1) for high supply pressures, maximum stiffness occurs at values of restrictor coefficient where the damping is minimum; thus, a compromise must be made between the two extremes. Whereas at low supply pressures maximum stiffness can be attained without sacrificing damping contributions;

(2) for low ε maximum stiffness occurs near the restrictor coefficient value where the mass flow through the orifices is critical. In the case of high ε maximum stiffness occurs at the restrictor coefficient values that gives critical mass flow;

(3) actual stiffness can be improved by operating at higher supply pressures because nondimensional stiffness is normalized by (P_s-1).

4.1.4 Higher Harmonics

The second and third harmonics of the load coefficients are tabulated according to the restrictor coefficient, supply pressure, and amplitude of oscillation and they are listed in Appendix B.

CHAPTER V

CONCLUSIONS AND OPTIMUM DESIGN

For the range of amplitudes considered in this work, the non-dimensional design parameters load capacity, stiffness and damping are not critically affected by the amplitude of the disturbances. Only in the range of low restrictor coefficient do the bearing design parameters show a small change with ε; however, for the majority of restrictor coefficients the curves appear to fall on top of each other. This is a very convenient feature since only one curve could be used to represent each relationship for the range of amplitude considered. The actual stiffness is little affected by the displacement amplitude, at least up to $\varepsilon = 0.5$. The actual damping varies inversely with $(1-\varepsilon^2)^{1.5}$.

In order to use the results presented in Chapter IV for bearing design purposes, the restrictor coefficient must be known. Thus, a choice between optimum stiffness and optimum damping must be made so that the restrictor coefficient can be identified. In general, to insure that there is no bearing surface contact, stiffness is selected first since working film thicknesses are of the order of one-thousandth. Although the choice of stiffness governs, some damping is necessary to cushion sudden load disturbances.

1. For maximum stiffness select a restrictor coefficient, $\Lambda = 0.8-2.5$. Higher supply pressures increase stiffness at expense of a greater mass flow rate.
2. In selecting damping the minimum allowable stiffness must be specified so the allowed damping working range can be determined. Low supply pressures provide high damping for low restrictor coefficient whereas the stiffness is considerably low. At high supply pressures damping improves when moving toward large restrictor coefficients from the point of maximum stiffness. Damping is insensitive to the supply pressure for high restrictor coefficients; however, higher supply pressures improve the corresponding stiffness.
3. Once the desirable stiffness and damping are established the restrictor coefficient is automatically fixed which in turn fixes the load capacity. Knowing the load capacity, the choice of supply pressure determines the bearing dimensions. Since the dimensionless stiffness and damping depend on the film thickness, the actual stiffness and damping can be improved by selecting smaller film thicknesses; however, the smallest film thickness that can be used is limited by two factors: (1) bearing surface roughness, (2) machining imperfections.

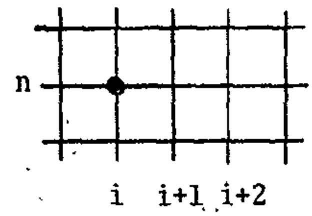
APPENDIX A

FINITE DIFFERENCE EXPRESSIONS AND DERIVATION OF DIFFERENCE EQUATIONS (3-1), (3-5), (3-6), (3-7) AND (3-8)

The finite difference expressions used to represent the various derivatives in this study are given below.

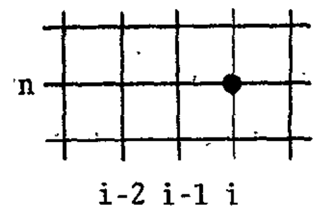
First spatial derivative, foward differences

$$\left.\frac{\partial P^2}{\partial x}\right|_i \approx - \frac{P^2_{i+2,n} + 4P^2_{i+1,n} - 3P^2_{i,n}}{2\ \Delta x} \qquad \text{(A-1)}$$



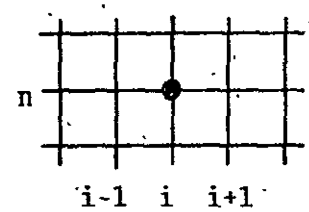
First spatial derivative, backward differences

$$\left.\frac{\partial P^2}{\partial x}\right|_i \approx - \frac{3P^2_{i,n} - 4P^2_{i-1,n} + P^2_{i-2,n}}{2\ \Delta x} \qquad \text{(A-2)}$$



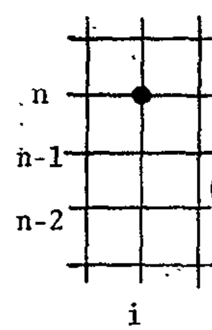
Second spatial derivative, central differences

$$\left.\frac{\partial^2 (P^2)}{\partial x^2}\right|_i \approx \frac{P^2_{i+1,n} - 2P^2_{i,n} + P^2_{i-1,n}}{\Delta x^2} \qquad \text{(A-3)}$$



First time derivative, backward differences

$$\left.\frac{\partial (Ph)}{\partial t}\right|_n \approx \frac{3P_{i,n}\, h_{i,n} - 4P_{i,n-1}\, h_{i,n-1} + P_{i,n-2}\, h_{i,n-2}}{2\ \Delta t} \qquad \text{(A-4)}$$



The implicit formulation from which the iterative equation (3-1) is derived, is obtained by substituting the difference expressions (A-3) and (A-4) into the Reynolds' equation in its nondimensional form, equation (2-2), and it is written as follows

$$\frac{3P_{i,n}\,h_{i,n} - 4P_{i,n-1}\,h_{i,n-1} + P_{i,n-2}\,h_{i,n-2}}{2\,\Delta t}$$

$$= \frac{h^3_{i,n}}{2\sigma}\;\frac{P^2_{i+1,n} - 2P^2_{i,n} + P^2_{i-1,n}}{\Delta x^2} \tag{A-5}$$

Rearranging and grouping of like terms

$$P_{i,n}\,[\,3 + 2\alpha P_{i,n}\,] = \alpha\,[\,P^2_{i+1,n} + P^2_{i-1,n}\,]$$

$$+ 4\beta P_{i,n-1} - \gamma P_{i,n-2} \tag{A-6}$$

where $\alpha = \dfrac{h_{i,n}}{\Delta x^2\sigma}$

$$\beta = \frac{h_{i,n-1}}{h_{i,n}}$$

$$\gamma = \frac{h_{i,n-2}}{h_{i,n}}$$

Since $P_{i,n}$ can not be solved explicitly from the foregoing equation, it was decided to set up an iterative procedure, equation (3-1), by applying the point S.O.R. method to the equation (A-6), and it can be written as

$$P_i^{(s+1)} = \frac{\alpha}{3 + 2\alpha P^{(s)}}\,[\,P^{2(s)}_{i+1} + P^{2(s+1)}_{i-1}\,]$$

$$+ 4\beta P_{i,n-1} - \gamma P_{i,n-2} \tag{A-7}$$

where (s) is an iteration counter.

The difference equations (3-5), (3-6), (3-7), and (3-8) are obtained by appropriately substituting the difference expressions (A-1) and (A-2) for the first derivatives that appear on each one of the equations (2-22), (2-23), (2-24), and (2-25); thus, equation (2-22) becomes

$$\frac{3P_{i,n}^{2} - 4P_{i-1,n}^{2} + P_{i-2,n}^{2}}{2\,\Delta x} - \frac{-P_{i+2,n}^{2} + 4P_{i+1,n}^{2} - 3P_{i,n}^{2}}{2\,\Delta x}$$

$$= F\left[\frac{P_{i,n}}{P_s}\right]^{1/k}\left[1 - \left[\frac{P_{i,n}}{P_s}\right]^{\frac{k-1}{k}}\right]^{1/2} \qquad \text{(A-8)}$$

Rearranging

$$P_{i,n} = \left\{\frac{2}{3}\left(P_{i+1,n}^{2} + P_{i-1,n}^{2}\right) - \left(P_{i+2,n}^{2} + P_{i-2,n}^{2}\right)/6 + \frac{\Delta x F}{3}\left[\frac{P_{i,n}}{P_s}\right]^{1/k}\left[1 - \left[\frac{P_{i,n}}{P_s}\right]^{\frac{k-1}{k}}\right]^{1/2}\right\}^{1/2} \qquad \text{(A-9)}$$

Since $P_{i,n}$ can not be solved explicitly the equation above is iteratively converged to a desired tolerance. The variable F is defined as

$$F = \frac{\Lambda\, P_s^{2}}{(1 + \varepsilon \sin(t))(1-a)}$$

Likewise equation (2-23) becomes

$$\frac{3P_{i,n}^2 - 4P_{i-1,n}^2 + P_{i-2,n}^2}{2\,\Delta x} - \frac{-P_{i+2,n}^2 + 4P_{i+1,n}^2 - 3P_{i,n}^2}{2\,\Delta x}$$

$$= F\left[\frac{k-1}{k+1}\right]^{1/2}\left[\frac{2}{k+1}\right]^{\frac{1}{k-1}} \qquad \text{(A-10)}$$

Solving for $P_{i,n}$ explicitly

$$P_{i,n} = \left\{\frac{2}{3}\left(P_{i+1,n}^2 + P_{i-1,n}^2\right) - \left(P_{i+2,n}^2 + P_{i-2,n}^2\right)/6 + \frac{\Delta x\ F}{3}\left[\frac{k-1}{k+1}\right]^{1/2}\left[\frac{2}{k+1}\right]^{\frac{1}{k-1}}\right\}^{1/2} \qquad \text{(A-11)}$$

Equation (2-24) is approximated as follows

$$\frac{-P_{i+2,n}^2 + 4P_{i+1,n}^2 - 3P_{i,n}^2}{2\,\Delta x} - \frac{3P_{i,n}^2 - 4P_{i-1,n}^2 + P_{i-2,n}^2}{2\,\Delta x}$$

$$= F\left[\frac{P_{i,n}}{P_s}\right]^{\frac{k-1}{k}}\left[1 - \left[\frac{P_s}{P_{i,n}}\right]^{\frac{k-1}{k}}\right]^{1/2} \qquad \text{(A-12)}$$

Rearranging

$$P_{i,n} = \left\{\frac{2}{3}\left(P_{i+1,n}^2 + P_{i-1,n}^2\right) - \left(P_{i+2,n}^2 + P_{i-2,n}^2\right)/6 - \frac{\Delta x\ F}{3}\left[\frac{P_{i,n}}{P_s}\right]^{\frac{k-1}{k}}\left[1 - \left[\frac{P_s}{P_{i,n}}\right]^{\frac{k-1}{k}}\right]^{1/2}\right\}^{1/2}$$

(A-13)

Equation (A-13) must be solved using an iterative procedure since $P_{i,n}$ is given implicitly.

Equation (2-25) is approximated as follows

$$\frac{-P_{i+2,n}^2 + 4P_{i+1,n}^2 - 3P_{i,n}^2}{2\,\Delta x} - \frac{3P_{i,n}^2 - 4P_{i-1,n}^2 + P_{i-2,n}^2}{2\,\Delta x}$$

$$= F\ \frac{P_{i,n}}{P_s}\left[\frac{k-1}{k+1}\right]^{1/2}\left[\frac{2}{k+1}\right]^{\frac{1}{k-1}} . \qquad \text{(A-14)}$$

Equation (A-14) is solved using the quadratic equation formula, and it can be written as

$$P_{i,n} = \frac{1}{12}\left[F1^2 + 24\left(4\left(P_{i+1,n}^2 + P_{i-1,n}^2\right) - \left(P_{i+2,n}^2 + P_{i-2,n}^2\right)\right)\right]^{1/2} - \frac{F1}{12}$$

$$\text{(A-15)}$$

where

$$F1 = \frac{2\Delta x\ F}{P_s}\left[\frac{k-1}{k+1}\right]^{1/2}\left[\frac{2}{k+1}\right]^{\frac{1}{k-1}}$$

The positive sign is selected to insure that the pressure magnitude stays positive during the calculations.

APPENDIX B

LOAD COEFFICIENTS TABLES

TABLE B.1

Load Coefficients for $P_s = 10.0$

($\epsilon = 0.1, \sigma = 0.1, a = 0.5$)

Λ	A_0	B_1	A_1	B_2	A_2	B_3	A_3
0.1	.700278+00	-.109477+00	-.122900-02	.132496-03	-.608717-02	.325668-03	.129846-03
0.2	.117607+01	-.169299+00	-.736110-03	.668326-04	-.916404-02	.472743-03	.762040-04
0.3	.156278+01	-.214732+00	-.525605-03	.435202-04	-.114306-01	.582857-03	.535061-04
0.5	.219674+01	-.285886+00	-.334971-03	.248456-04	-.149453-01	.755488-03	.334684-04
0.7	.272316+01	-.343098+00	-.245009-03	.179219-04	-.177644-01	.894709-03	.246513-04
0.8	.295999+01	-.368471+00	-.215419-03	.155232-04	-.190155-01	.957179-03	.215325-04
0.9	.318316+01	-.391889+00	-.196165-03	.211998-04	-.198821-01	.774341-03	.280589-04
1.0	.339312+01	-.410840+00	-.211479-03	.673878-04	-.186914-01	-.186300-03	.603174-04
1.1	.359029+01	-.424359+00	-.258555-03	.125530-03	-.163627-01	-.396016-03	.702608-04
1.2	.377532+01	-.432975+00	-.327742-03	.166638-03	-.144727-01	.344019-04	.639707-04
1.3	.394865+01	-.438028+00	-.406525-03	.186254-03	-.135093-01	.601745-04	.701750-04
1.5	.426230+01	-.442804+00	-.553507-03	.214218-03	-.116977-01	-.102399-03	.859320-04
1.6	.440443+01	-.443108+00	-.620204-03	.224796-03	-.107446-01	-.178449-03	.930518-04
1.7	.453786+01	-.442260+00	-.682370-03	.233942-03	-.977701-02	-.249461-03	.993791-04
2.0	.489243+01	-.434231+00	-.842049-03	.251938-03	-.687944-02	-.431613-03	.114363-03
2.5	.536158+01	-.408910+00	-.102782-02	.260586-03	-.245195-02	-.623517-03	.128035-03
2.8	.558685+01	-.389847+00	-.110184-02	.255552-03	-.185144-03	-.678956-03	.133420-03
3.0	.571839+01	-.376372+00	-.113769-02	.251765-03	.114052-02	-.695388-03	.134326-03
3.5	.599437+01	-.341894+00	-.119436-02	.236539-03	.382042-02	-.681659-03	.136977-03
4.0	.621093+01	-.308316+00	-.121204-02	.215430-03	.568193-02	-.617466-03	.134439-03
5.0	.652154+01	-.248667+00	-.118976-02	.178981-03	.757377-02	-.437595-03	.128230-03
7.0	.686755+01	-.163570+00	-.106885-02	.133320-03	.764848-02	-.149811-03	.114222-03
10.0	.709601+01	-.946707-01	-.912842-03	.109720-03	.567538-02	.140381-04	.994407-04
15.0	.723630+01	-.466006-01	-.779206-03	.103445-03	.327134-02	.459915-04	.869484-04

TABLE B.2

Load Coefficients for $P_s = 6.0$

($\varepsilon = 0.1$, $\sigma = 0.1$, $a = 0.5$)

Λ	A_0	B_1	A_1	B_2	A_2	B_3	A_3
0.1	.301476+00	-.526508-01	-.218183-02	.301598-03	-.287293-02	.169942-03	.241142-03
0.2	.539487+00	-.876193-01	-.151790-02	.177669-03	-.490392-02	.268902-03	.162519-03
0.3	.743350+00	-.115155+00	-.116605-02	.123262-03	-.638746-02	.339972-03	.122823-03
0.5	.108990+01	-.158859+00	-.799673-03	.743603-04	-.863693-02	.447282-03	.825074-04
0.7	.138519+01	-.194121+00	-.609193-03	.522053-04	-.104059-01	.532780-03	.621911-04
0.8	.151958+01	-.209747+00	-.544051-03	.456073-04	-.111783-01	.565431-03	.560891-04
0.9	.164647+01	-.223371+00	-.508793-03	.659337-04	-.111515-01	.161273-03	.743929-04
1.0	.176589+01	-.233802+00	-.511187-03	.111265-03	-.101240-01	-.140382-03	.884999-04
1.1	.187814+01	-.241127+00	-.540224-03	.151648-03	-.905573-02	.315416-04	.850252-04
1.2	.198349+01	-.245991+00	-.588447-03	.173137-03	-.848024-02	.826864-04	.864769-04
1.3	.208208+01	-.249434+00	-.639523-03	.188443-03	-.799454-02	.304055-04	.928561-04
1.5	.226101+01	-.253189+00	-.740619-03	.213341-03	-.692161-02	-.692213-04	.104900-03
1.6	.234232+01	-.253766+00	-.788550-03	.223515-03	-.635531-02	-.115530-03	.109954-03
1.7	.241876+01	-.253626+00	-.835115-03	.232158-03	-.577859-02	-.158556-03	.114944-03
2.0	.262236+01	-.249779+00	-.956397-03	.250244-03	-.405182-02	-.267688-03	.126220-03
2.5	.289257+01	-.235891+00	-.110307-02	.257280-03	-.141273-02	-.381971-03	.136567-03
2.8	.302256+01	-.225108+00	-.116074-02	.253783-03	-.657852-04	-.414648-03	.139388-03
3.0	.309851+01	-.217423+00	-.118844-02	.247982-03	.721780-03	-.423293-03	.140905-03
3.5	.325796+01	-.197631+00	-.122914-02	.232256-03	.230760-02	-.413388-03	.140814-03
4.0	.338313+01	-.178261+00	-.123434-02	.212631-03	.340459-02	-.374174-03	.136834-03
5.0	.356269+01	-.143757+00	-.119947-02	.176796-03	.451064-02	-.265332-03	.129511-03
7.0	.376263+01	-.944872-01	-.106904-02	.132505-03	.453780-02	-.939928-04	.114824-03
10.0	.389451+01	-.546363-01	-.910493-03	.109603-03	.337211-02	.196410-05	.994911-04
15.0	.397543+01	-.268759-01	-.776858-03	.103204-03	.196526-02	.202014-04	.870014-04

TABLE B.3

Load Coefficients for $P_s = 4.0$

($\epsilon = 0.1$, $\sigma = 0.1$, $a = 0.5$)

Λ	A_0	B_1	A_1	B_2	A_2	B_3	A_3
0.1	.145641+00	-.271348-01	-.289259-02	.465243-03	-.120681-02	.828230-04	.330626-03
0.2	.272012+00	-.480234-01	-.229480-02	.325186-03	-.258522-02	.155842-03	.254903-03
0.3	.385759+00	-.654791-01	-.189506-02	.246259-03	-.364399-02	.207864-03	.204862-03
0.5	.587191+00	-.942243-01	-.141488-02	.162151-03	-.526623-02	.286048-03	.148952-03
0.7	.764465+00	-.117904+00	-.113086-02	.119692-03	-.652070-02	.336050-03	.118094-03
0.8	.846019+00	-.127757+00	-.104206-02	.125963-03	-.657313-02	.114638-03	.123823-03
0.9	.922948+00	-.135501+00	-.991801-03	.149103-03	-.614351-02	.511541-04	.124323-03
1.0	.995380+00	-.141288+00	-.971776-03	.167749-03	-.580810-02	.121364-03	.119528-03
1.1	.106338+01	-.145707+00	-.969684-03	.180184-03	-.559976-02	.870472-04	.121292-03
1.2	.112715+01	-.149165+00	-.983357-03	.191492-03	-.534288-02	.521457-04	.125680-03
1.3	.118701+01	-.151784+00	-.991198-03	.203273-03	-.504646-02	.164206-04	.126347-03
1.5	.129614+01	-.154923+00	-.103043-02	.222469-03	-.437659-02	-.501530-04	.131928-03
1.6	.134593+01	-.155614+00	-.105291-02	.230295-03	-.401742-02	-.807783-04	.134583-03
1.7	.139282+01	-.155818+00	-.107573-02	.236870-03	-.365000-02	-.109531-03	.136958-03
2.0	.151812+01	-.154095+00	-.114166-02	.250212-03	-.254152-02	-.181983-03	.142591-03
2.5	.168508+01	-.146085+00	-.122563-02	.254070-03	-.840756-03	-.256522-03	.147608-03
2.8	.176560+01	-.139573+00	-.125705-02	.248946-03	.265899-04	-.276625-03	.147979-03
3.0	.181268+01	-.134874+00	-.127079-02	.243516-03	.532111-03	-.281802-03	.147535-03
3.5	.191159+01	-.122666+00	-.128368-02	.225978-03	.154599-02	-.273195-03	.144605-03
4.0	.198926+01	-.110640+00	-.127358-02	.206686-03	.224189-02	-.245954-03	.140497-03
5.0	.210064+01	-.891517-01	-.121594-02	.171810-03	.293162-02	-.174010-03	.131159-03
7.0	.222449+01	-.584740-01	-.106847-02	.130303-03	.292345-02	-.640128-04	.114845-03
10.0	.230601+01	-.337395-01	-.905115-03	.108873-03	.217168-02	-.384348-05	.994567-04
15.0	.235592+01	-.165713-01	-.771997-03	.103196-03	.128268-02	.704238-05	.868350-04

TABLE B.4

Load Coefficients for $P_s = 2.0$

(ϵ = 0.1, σ = 0.1, a = 0.5)

Λ	A_0	B_1	A_1	B_2	A_2	B_3	A_3
0.1	.392426-01	-.783403-02	-.363130-02	.663595-03	.224491-03	.217790-06	.436207-03
0.2	.759825-01	-.146472-01	-.333665-02	.578079-03	-.290756-03	.298010-04	.392898-03
0.3	.110972+00	-.207621-01	-.308607-02	.510684-03	-.713248-03	.499641-04	.358709-03
0.5	.175716+00	-.309729-01	-.269011-02	.415513-03	-.130131-02	.681484-04	.307174-03
0.7	.233751+00	-.387386-01	-.240742-02	.359090-03	-.160535-02	.627230-04	.273789-03
0.8	.260406+00	-.418165-01	-.229733-02	.339975-03	-.167278-02	.546402-04	.260817-03
0.9	.285579+00	-.444197-01	-.220650-02	.325644-03	-.169593-02	.440097-04	.251388-03
1.0	.309347+00	-.465921-01	-.212850-02	.314564-03	-.168140-02	.315276-04	.243140-03
1.1	.331781+00	-.483759-01	-.206206-02	.305913-03	-.163593-02	.180994-04	.236260-03
1.2	.352955+00	-.498105-01	-.200514-02	.299486-03	-.156570-02	.480152-05	.229947-03
1.3	.372939+00	-.509329-01	-.195537-02	.294083-03	-.147555-02	-.946195-05	.224626-03
1.5	.409605+00	-.523722-01	-.187546-02	.285148-03	-.125278-02	-.359056-04	.215847-03
1.6	.426415+00	-.527486-01	-.184326-02	.281532-03	-.112751-02	-.485457-04	.212439-03
1.7	.442289+00	-.529313-01	-.181364-02	.277529-03	-.996627-03	-.600380-04	.209002-03
2.0	.484838+00	-.525410-01	-.173941-02	.266278-03	-.593930-03	-.880504-04	.199429-03
2.5	.541670+00	-.498104-01	-.163983-02	.245115-03	.278599-04	-.114120-03	.185220-03
2.8	.569043+00	-.474841-01	-.158715-02	.231459-03	.339053-03	-.119174-03	.177408-03
3.0	.585018+00	-.457921-01	-.155218-02	.222378-03	.516897-03	-.119250-03	.172108-03
3.5	.618451+00	-.413902-01	-.146833-02	.200017-03	.860538-03	-.111259-03	.160199-03
4.0	.644541+00	-.370787-01	-.139120-02	.179907-03	.107967-02	-.971891-04	.150369-03
5.0	.681602+00	-.294866-01	-.125251-02	.149353-03	.125970-02	-.663663-04	.134414-03
7.0	.722150+00	-.189569-01	-.104632-02	.118488-03	.116358-02	-.264712-04	.113518-03
10.0	.748339+00	-.107585-01	-.870333-02	.105971-03	.848787-03	-.838591-05	.961900-04
15.0	.764132+00	-.522702-02	-.747599-02	.103501-03	.526888-03	-.684941-05	.849319-04

TABLE B.5

Load Coefficients for $P_s = 1.5$

$(\varepsilon = 0.1,\ \sigma = 0.1,\ a = 0.5)$

Λ	A_0	B_1	A_1	B_2	A_2	B_3	A_3
0.1	.214437-01	-.441711-02	-.370847-02	.677821-03	.435032-03	.526073-04	.423215-03
0.2	.409595-01	-.801719-02	-.350180-02	.623355-03	.195434-03	.842959-04	.379095-03
0.3	.594628-01	-.111413-01	-.331818-02	.568652-03	-.503208-04	.606254-04	.367713-03
0.4	.153799-01	-.138706-01	-.318708-02	.538598-03	-.172227-03	.131290-04	.374131-03
0.5	.933780-01	-.163011-01	-.301846-02	.501724-03	-.305366-03	.175841-04	.347410-03
0.8	.137307+00	-.216799-01	-.267439-02	.420323-03	-.495723-03	.645134-05	.301361-03
0.9	.150259+00	-.229273-01	-.258212-02	.400143-03	-.504703-03	.585147-06	.289987-03
1.0	.162439+00	-.239434-01	-.249878-02	.382463-03	-.493768-03	-.608893-05	.279752-03
1.1	.173889+00	-.247513-01	-.242289-02	.366734-03	-.466732-03	-.131164-04	.270627-03
1.2	.184651+00	-.253738-01	-.237487-02	.351838-03	-.426369-03	-.197301-04	.269123-03
1.3	.194766+00	-.258303-01	-.230992-02	.338719-03	-.376241-03	-.269047-04	.260898-03
1.5	.213206+00	-.263216-01	-.219512-02	.315558-03	-.255936-03	-.403162-04	.246785-03
1.6	.221604+00	-.263894-01	-.212543-02	.306325-03	-.189913-03	-.468471-04	.234512-03
1.7	.229500+00	-.263596-01	-.207734-02	.296317-03	-.121913-03	-.518905-04	.228639-03
2.0	.250480+00	-.258071-01	-.194783-02	.269300-03	.811268-04	-.635644-04	.212691-03
2.5	.278021+00	-.239240-01	-.178064-02	.229635-03	.373783-03	-.704024-04	.194922-03
2.8	.291057+00	-.225180-01	-.168525-02	.209827-03	.508339-03	-.693794-04	.183059-03
3.0	.298587+00	-.215403-01	-.161613-02	.199566-03	.580276-03	-.673099-04	.172447-03
3.5	.314145+00	-.191091-01	-.148763-02	.176636-03	.705790-03	-.589231-04	.157906-03
4.0	.326079+00	-.168401-01	-.138387-02	.154548-03	.769294-03	-.490760-04	.148162-03
5.0	.342679+00	-.130429-01	-.120846-02	.129846-03	.783537-03	-.322804-04	.129989-03
7.0	.360305+00	-.810851-02	-.987669-03	.109789-03	.657618-03	-.151105-04	.108447-03
10.0	.371350+00	-.449230-02	-.824589-03	.104095-03	.473221-03	-.974558-05	.922709-04
15.0	.377868+00	-.215309-02	-.720866-03	.103903-03	.315006-03	-.106243-04	.825500-04

TABLE B.6

Load Coefficients for P_s = 10.0

(ε = 0.3, σ = 0.1, a = 0.5)

Λ	A_0	B_1	A_1	B_2	A_2	B_3	A_3
0.1	.755600+00	-.354572+00	-.385419-02	.111049-02	-.618645-01	.994390-02	.564165-03
0.2	.125647+01	-.545054+00	-.228977-02	.561363-03	-.910567-01	.142774-01	.308925-03
0.3	.166205+01	-.689735+00	-.163298-02	.369111-03	-.112846+00	.175451-01	.213132-03
0.5	.232560+01	-.916452+00	-.103908-02	.215037-03	-.146879+00	.226923-01	.131756-03
0.7	.287034+01	-.108830+01	-.926743-03	.431602-03	-.165404+00	.202694-01	.416796-03
0.8	.310860+01	-.115249+01	-.103481-02	.708597-03	-.165242+00	.151260-01	.664106-03
0.9	.332796+01	-.120377+01	-.120560-02	.101398-02	-.160484+00	.968732-02	.867208-03
1.0	.353071+01	-.124366+01	-.140741-02	.130753-02	-.152538+00	.491750-02	.101234-02
1.1	.371871+01	-.127355+01	-.161367-02	.157660-02	-.142580+00	.118964-02	.111488-02
1.2	.389353+01	-.129474+01	-.182755-02	.181226-02	-.131517+00	-.150071-02	.116977-02
1.3	.405648+01	-.130839+01	-.204592-02	.200863-02	-.120046+00	-.335494-02	.119099-02
1.5	.435108+01	-.131719+01	-.245465-02	.229044-02	-.977546-01	-.582116-02	.119480-02
1.6	.448452+01	-.131415+01	-.264246-02	.238345-02	-.874618-01	-.701851-02	.118902-02
1.7	.460969+01	-.130726+01	-.281629-02	.244838-02	-.778515-01	-.844092-02	.118602-02
2.0	.494093+01	-.127132+01	-.324700-02	.254271-02	-.513347-01	-.127271-01	.116487-02
2.5	.537609+01	-.118382+01	-.369423-02	.250867-02	-.132118-01	-.167154-01	.106956-02
2.8	.558463+01	-.112385+01	-.384483-02	.242617-02	.534968-02	-.175572-01	.100093-02
3.0	.570657+01	-.108291+01	-.390995-02	.235736-02	.159382-01	-.176364-01	.955935-03
3.5	.596358+01	-.981511+00	-.398390-02	.216544-02	.367316-01	-.166688-01	.857134-03
4.0	.616731+01	-.885666+00	-.397324-02	.197721-02	.506914-01	-.147349-01	.778834-03
5.0	.646524+01	-.718925+00	-.382989-02	.166372-02	.643227-01	-.101492-01	.680519-03
7.0	.681129+01	-.482255+00	-.343920-02	.130209-02	.639308-01	-.331797-02	.608458-03
10.0	.705407+01	-.286150+00	-.299288-02	.112360-02	.479422-01	.538159-03	.502523-03
15.0	.721230+01	-.144206+00	-.260760-02	.106165-02	.278415-01	.143463-02	.566353-03

TABLE B.7

Load Coefficients for $P_s = 6.0$

($\varepsilon = 0.3$, $\sigma = 0.1$, $a = 0.5$)

Λ	A_0	B_1	A_1	B_2	A_2	B_3	A_3
0.1	.331654+00	-.173073+00	-.698407-02	.256379-02	-.321375-01	.551675-02	.122257-02
0.2	.585405+00	-.284900+00	-.478190-02	.149336-02	-.508172-01	.831078-02	.737041-03
0.3	.801083+00	-.372659+00	-.365295-02	.103304-02	-.646864-01	.103609-01	.528859-03
0.5	.116594+01	-.511772+00	-.249226-02	.627618-03	-.859818-01	.134875-01	.343499-03
0.7	.147115+01	-.615221+00	-.209003-02	.770534-03	-.957891-01	.111025-01	.607789-03
0.8	.160568+01	-.653653+00	-.208460-02	.994109-03	-.953390-01	.820732-02	.801355-03
0.9	.173001+01	-.684393+00	-.214829-02	.124047-02	-.924395-01	.537387-02	.958463-03
1.0	.184526+01	-.708368+00	-.225513-02	.148225-02	-.878737-01	.299354-02	.106916-02
1.1	.195239+01	-.726433+00	-.238697-02	.170254-02	-.822799-01	.117221-02	.113832-02
1.2	.205219+01	-.739367+00	-.253143-02	.189497-02	-.761553-01	-.165002-03	.117758-02
1.3	.214535+01	-.747873+00	-.268316-02	.205475-02	-.698669-01	-.117862-02	.119796-02
1.5	.231403+01	-.754081+00	-.298105-02	.228617-02	-.577043-01	-.294999-02	.120897-02
1.6	.239049+01	-.752904+00	-.312165-02	.236161-02	-.520381-01	-.395991-02	.121102-02
1.7	.246218+01	-.749614+00	-.325187-02	.241647-02	-.465841-01	-.502338-02	.121188-02
2.0	.265196+01	-.731003+00	-.357215-02	.250177-02	-.308591-01	-.760572-02	.118260-02
2.5	.290202+01	-.682524+00	-.390202-02	.246341-02	-.822498-02	-.996496-02	.107908-02
2.8	.302210+01	-.648565+00	-.400645-02	.238074-02	.277869-02	-.104509-01	.100868-02
3.0	.309238+01	-.625224+00	-.404686-02	.231335-02	.904974-02	-.104877-01	.962251-03
3.5	.324061+01	-.567092+00	-.407477-02	.212643-02	.213443-01	-.989096-02	.860807-03
4.0	.335821+01	-.511897+00	-.403422-02	.194285-02	.295790-01	-.872861-02	.781957-03
5.0	.353030+01	-.415587+00	-.385589-02	.163768-02	.375912-01	-.600319-02	.683040-03
7.0	.373023+01	-.278657+00	-.343880-02	.128846-02	.373396-01	-.198531-02	.610354-03
10.0	.387044+01	-.165262+00	-.298039-02	.111208-02	.279569-01	.292873-02	.583083-03
15.0	.396171+01	-.832277-01	-.259956-02	.106055-02	.163017-01	.770444-03	.566618-03

TABLE B.8

Load Coefficients for $P_s = 4.0$

($\varepsilon = 0.3$, $\sigma = 0.1$, $a = 0.5$)

Λ	A_0	B_1	A_1	B_2	A_2	B_3	A_3
0.1	.163214+00	-.907109-01	-.949267-02	.404539-02	-.170244-01	.314288-02	.192810-02
0.2	.299998+00	-.158208+00	-.736843-02	.277297-02	-.295192-01	.511588-02	.131929-02
0.3	.421856+00	-.214182+00	-.604699-02	.208064-02	-.391872-01	.658099-02	.100192-02
0.5	.635646+00	-.305321+00	-.449456-02	.140050-02	-.536140-01	.833094-02	.728624-03
0.7	.817896+00	-.371494+00	-.381450-02	.139338-02	-.586387-01	.619093-02	.894210-03
0.8	.898992+00	-.396004+00	-.366872-02	.151086-02	-.581437-01	.470900-02	.997154-03
0.9	.974287+00	-.415639+00	-.359858-02	.165360-02	-.563731-01	.340471-02	.108046-02
1.0	.104434+01	-.431005+00	-.358016-02	.180018-02	-.537696-01	.234331-02	.114104-02
1.1	.110964+01	-.442657+00	-.359753-02	.193827-02	-.506749-01	.147495-02	.118137-02
1.2	.117059+01	-.451104+00	-.363862-02	.205899-02	-.473373-01	.701034-03	.120707-02
1.3	.122758+01	-.456810+00	-.369650-02	.216159-02	-.439225-01	-.784286-04	.122622-02
1.5	.133082+01	-.461747+00	-.382750-02	.231089-02	-.370730-01	-.176373-02	.124844-02
1.6	.137758+01	-.461766+00	-.389223-02	.236347-02	-.335909-01	-.254070-02	.124971-02
1.7	.142143+01	-.460490+00	-.395369-02	.240316-02	-.301112-01	-.324768-02	.124525-02
2.0	.153785+01	-.450700+00	-.410729-02	.245770-02	-.200204-01	-.493954-02	.120334-02
2.5	.169183+01	-.422278+00	-.425206-02	.239724-02	-.546180-02	-.646189-02	.108812-02
2.8	.176597+01	-.401733+00	-.428027-02	.230943-02	.160916-02	-.676181-02	.101347-02
3.0	.180941+01	-.387479+00	-.428014-02	.224116-02	.563162-02	-.677491-02	.966394-03
3.5	.190113+01	-.351712+00	-.423059-02	.205788-02	.134963-01	-.636346-02	.862319-03
4.0	.197397+01	-.317548+00	-.413829-02	.188038-02	.187363-01	-.559495-02	.783205-03
5.0	.208061+01	-.257720+00	-.389967-02	.158941-02	.237846-01	-.382737-02	.685091-03
7.0	.220448+01	-.172555+00	-.343692-02	.126296-02	.235378-01	-.127201-02	.612440-03
10.0	.229122+01	-.102153+00	-.296716-02	.110068-02	.175795-01	.145260-03	.585445-03
15.0	.234757+01	-.513789+01	-.258638-02	.105745-02	.102763-01	.428652-03	.567371-03

TABLE B.9

Load Coefficients for $P_s = 2.0$

($\varepsilon = 0.3$, $\sigma = 0.1$, $a = 0.5$)

Λ	A_0	B_1	A_1	B_2	A_2	B_3	A_3
0.1	.460333-01	-.273052-01	-.122292-01	.595160-02	-.402019-02	.919357-03	.290329-02
0.2	.865427-01	-.495765-01	-.110938-01	.511284-02	-.857548-02	.165223-02	.246478-02
0.3	.124479+00	-.690737-01	-.101636-01	.447768-02	-.121382-01	.211577-02	.215106-02
0.5	.192935+00	-.100354+00	-.879192-02	.365762-02	-.166424-01	.238083-02	.178383-02
0.7	.252391+00	-.122784+00	-.788048-02	.321769-02	-.184084-01	.204167-02	.161320-02
0.8	.279117+00	-.131240+00	-.754130-02	.308114-02	-.185351-01	.173736-02	.156409-02
0.9	.304038+00	-.138137+00	-.725978-02	.297883-02	-.182812-01	.138177-02	.152801-02
1.0	.327293+00	-.143661+00	-.702198-02	.290013-02	-.177250-01	.996984-03	.149820-02
1.1	.349012+00	-.147983+00	-.681853-02	.283747-02	-.169337-01	.600569-03	.147154-02
1.2	.369313+00	-.151256+00	-.664390-02	.278439-02	-.159638-01	.206567-03	.144591-02
1.3	.388310+00	-.153616+00	-.648952-02	.273794-02	-.148632-01	-.174706-03	.141948-02
1.5	.422784+00	-.156069+00	-.622923-02	.265237-02	-.124238-01	-.871715-03	.136432-02
1.6	.438442+00	-.156365+00	-.611640-02	.261088-02	-.111451-01	-.117839-02	.133472-02
1.7	.453153+00	-.156155+00	-.601206-02	.256862-02	-.985801-02	-.145427-02	.130381-02
2.0	.492294+00	-.153182+00	-.573451-02	.243664-02	-.610433-02	-.209342-02	.120780-02
2.5	.544186+00	-.143435+00	-.534445-02	.220456-02	-.726222-03	-.260245-02	.105330-02
2.8	.569179+00	-.136199+00	-.513333-02	.206837-02	.183239-02	-.265778-02	.972039-03
3.0	.583814+00	-.131153+00	-.500277-02	.198114-02	.326247-02	-.262329-02	.924731-03
3.5	.614673+00	-.118476+00	-.470110-02	.178317-02	.597782-02	-.238066-02	.828195-03
4.0	.639107+00	-.106405+00	-.443165-02	.161892-02	.768972-02	-.203023-02	.758985-03
5.0	.674692+00	-.854615-01	-.398544-02	.138159-02	.913055-02	-.131996-02	.677700-03
7.0	.715563+00	-.562557-01	-.337186-02	.115428-02	.853874-02	-.417429-03	.616182-03
10.0	.743708+00	-.327939-01	-.287159-02	.106341-02	.618959-02	.119158-04	.586573-03
15.0	.761679+00	-.163299-01	-.251457-02	.105070-02	.363179-02	.625293-04	.565428-03

TABLE B.10

Load Coefficients for $P_s = 1.5$

($\varepsilon = 0.3$, $\sigma = 0.1$, $a = 0.5$)

Λ	A_0	B_1	A_1	B_2	A_2	B_3	A_3
0.1	.259488-01	-.156276-01	-.126391-01	.630004-02	-.132791-02	.477311-03	.305584-02
0.2	.474243-01	-.271559-01	-.118576-01	.568113-02	-.371490-02	.767676-03	.276769-02
0.3	.673524-01	-.371770-01	-.111625-01	.518282-02	-.549835-02	.972681-03	.251300-02
0.4	.858488-01	-.457367-01	-.105609-01	.477356-02	-.679149-02	.105522-02	.231079-02
0.5	.103002+00	-.529632-01	-.100400-01	.443634-02	-.765755-02	.103848-02	.214982-02
0.8	.147321+00	-.679563-01	-.883093-02	.371692-02	-.832211-02	.609233-03	.181415-02
0.9	.160012+00	-.711355-01	-.851292-02	.354056-02	-.808255-02	.399741-03	.173244-02
1.0	.171798+00	-.735891-01	-.822592-02	.338450-02	-.769061-02	.180113-03	.165882-02
1.1	.182755+00	-.754124-01	-.796398-02	.324388-02	-.718403-02	-.394644-04	.159188-02
1.2	.192951+00	-.766914-01	-.772445-02	.311544-02	-.659431-02	-.251933-03	.152916-02
1.3	.202449+00	-.775021-01	-.750163-02	.299723-02	-.594817-02	-.451747-03	.146972-02
1.5	.219579+00	-.779820-01	-.710213-02	.278317-02	-.457020-02	-.799977-03	.136041-02
1.6	.227311+00	-.777627-01	-.692183-02	.268514-02	-.387030-02	-.944790-03	.130994-02
1.7	.234547+00	-.773002-01	-.675085-02	.259249-02	-.317890-02	-.106988-02	.126162-02
2.0	.253659+00	-.748245-01	-.628897-02	.234071-02	-.123262-02	-.133011-02	.113197-02
2.5	.278657+00	-.686508-01	-.565086-02	.199762-02	.137423-02	-.145693-02	.963854-03
2.8	.290545+00	-.644676-01	-.532957-02	.183169-02	.252711-02	-.141221-02	.888499-03
3.0	.297456+00	-.616485-01	-.513673-02	.173565-02	.313989-02	-.135275-02	.847252-03
3.5	.311893+00	-.548100-01	-.471657-02	.154024-02	.421656-02	-.115193-02	.769165-03
4.0	.323180+00	-.485425-01	-.437001-02	.139690-02	.479586-02	-.932232-03	.717503-03
5.0	.339353+00	-.381045-01	-.384230-02	.121819-02	.506724-02	-.559690-03	.658861-03
7.0	.357446+00	-.243237-01	-.320293-02	.107939-02	.431514-02	-.170579-03	.612213-03
10.0	.369520+00	-.138525-01	-.273909-02	.104346-02	.300441-02	-.297911-04	.583422-03
15.0	.377022+00	-.681990-02	-.243538-02	.105135-02	.179406-02	-.391021-04	.561925-03

TABLE B.11

Load Coefficients for P_s = 10.0

(ε = 0.5, σ = 0.1, a = 0.5)

Λ	A_0	B_1	A_1	B_2	A_2	B_3	A_3
0.1	.892562+00	-.700641+00	-.698354-02	.330376-02	-.211116+00	.587536-01	.158449-02
0.2	.145477+01	-.106419+01	-.406206-02	.166262-02	-.307595+00	.838854-01	.814917-03
0.3	.190618+01	-.133959+01	-.289815-02	.114842-02	-.379231+00	.102096+00	.627022-03
0.4	.228736+01	-.155529+01	-.266076-02	.162375-02	-.425915+00	.106473+00	.147556-02
0.5	.261510+01	-.172109+01	-.295415-02	.264852-02	-.448302+00	.980301-01	.272353-02
0.7	.315450+01	-.194276+01	-.404487-02	.486545-02	-.443437+00	.638269-01	.480880-02
0.8	.338059+01	-.201304+01	-.465923-02	.583053-02	-.425226+00	.444179-01	.545214-02
0.9	.358416+01	-.206319+01	-.517761-02	.661515-02	-.400829+00	.257744-01	.592952-02
1.0	.376883+01	-.209705+01	-.569323-02	.726196-02	-.372363+00	.860985-02	.617114-02
1.1	.393733+01	-.211763+01	-.609378-02	.777808-02	-.341591+00	-.664449-02	.632174-02
1.2	.409196+01	-.212736+01	-.647479-02	.814459-02	-.309649+00	-.198727-01	.631503-02
1.3	.423450+01	-.212822+01	-.679104-02	.845472-02	-.277534+00	-.311412-01	.627981-02
1.5	.448918+01	-.210942+01	-.730694-02	.877860-02	-.215120+00	-.483132-01	.600660-02
1.6	.460358+01	-.209216+01	-.750551-02	.883830-02	-.185659+00	-.546439-01	.582821-02
1.7	.471060+01	-.207091+01	-.768065-02	.884363-02	-.157682+00	-.597412-01	.563400-02
2.0	.499427+01	-.199025+01	-.802825-02	.869239-02	-.837831-01	-.697299-01	.504793-02
2.5	.537131+01	-.182830+01	-.827099-02	.809039-02	.762195-02	-.762949-01	.425393-02
2.8	.555413+01	-.172867+01	-.827664-02	.768240-02	.482330-01	-.755372-01	.387195-02
3.0	.566196+01	-.166354+01	-.824435-02	.742322-02	.705818-01	-.736448-01	.366027-02
3.5	.589238+01	-.150842+01	-.809652-02	.682801-02	.112841+00	-.660324-01	.324976-02
4.0	.607909+01	-.136671+01	-.785852-02	.635029-02	.139967+00	-.565728-01	.296203-02
5.0	.636146+01	-.112503+01	-.749287-02	.557040-02	.164915+00	-.379602-01	.268103-02
7.0	.671038+01	-.780585+00	-.683152-02	.474081-02	.160451+00	-.125251-01	.254699-02
10.0	.697688+01	-.483125+00	-.618159-02	.429346-02	.122907+00	.242425-02	.257353-02

TABLE B.12

Load Coefficients for P_s = 6.0

(ε = 0.5, σ = 0.1, a = 0.5)

Λ	A_0	B_1	A_1	B_2	A_2	B_3	A_3
0.1	.407335+00	-.352426+00	-.132372-01	.783786-02	-.114041+00	.336830-01	.390180-02
0.2	.699441+00	-.567444+00	-.877016-02	.446841-02	-.174890+00	.494072-01	.215095-02
0.3	.943337+00	-.734017+00	-.666060-02	.318371-02	-.219300+00	.601552-01	.163001-02
0.4	.115270+01	-.864351+00	-.578728-02	.319590-02	-.247216+00	.620481-01	.223233-02
0.5	.133469+01	-.964936+00	-.558990-02	.385395-02	-.260638+00	.568833-01	.324159-02
0.7	.163717+01	-.110039+01	-.600568-02	.556569-02	-.258253+00	.371638-01	.496724-02
0.8	.176480+01	-.114378+01	-.633689-02	.633399-02	-.247820+00	.261068-01	.553972-02
0.9	.188009+01	-.117510+01	-.668164-02	.698645-02	-.233734+00	.154783-01	.591019-02
1.0	.198490+01	-.119661+01	-.698795-02	.751338-02	-.217317+00	.569805-02	.613890-02
1.1	.208076+01	-.121014+01	-.727746-02	.792868-02	-.199524+00	-.301139-02	.623791-02
1.2	.216884+01	-.121714+01	-.750139-02	.824374-02	-.181087+00	-.106048-01	.624066-02
1.3	.225017+01	-.121881+01	-.773786-02	.847256-02	-.162537+00	-.171075-01	.618026-02
1.5	.239570+01	-.120979+01	-.807453-02	.871744-02	-.126546+00	-.271927-01	.591562-02
1.6	.246117+01	-.120056+01	-.820253-03	.875573-02	-.109571+00	-.310034-01	.574375-02
1.7	.252244+01	-.118893+01	-.830194-02	.875042-02	-.934667-01	-.341531-01	.555650-02
2.0	.268502+01	-.114390+01	-.849135-02	.855509-02	-.508928-01	-.407098-01	.500494-02
2.5	.290103+01	-.105290+01	-.856474-02	.794372-02	.265495-02	-.450972-01	.422913-02
2.8	.300588+01	-.996515+00	-.849715-02	.755263-02	.267166-02	-.446286-01	.385390-02
3.0	.306780+01	-.959465+00	-.840017-02	.731599-02	.399550-01	-.434923-01	.363728-02
3.5	.320029+01	-.870810+00	-.814256-02	.676238-02	.649789-01	-.389604-01	.321827-02
4.0	.330781+01	-.789460+00	-.793794-02	.625739-02	.810416-01	-.333496-01	.295662-02
5.0	.347062+01	-.650230+00	-.752417-02	.550428-02	.958304-01	-.223648-01	.268273-02
7.0	.367209+01	-.451228+00	-.683650-02	.469841-02	.933432-01	-.744193-02	.255451-02
10.0	.382601+01	-.279204+00	-.617032-02	.427634-02	.714929-01	.125636-02	.257949-02

TABLE B.13

Load Coefficients for $P_s = 4.0$

($\varepsilon = 0.5$, $\sigma = 0.1$, $a = 0.5$)

Λ	A_0	B_1	A_1	B_2	A_2	B_3	A_3
0.1	.207889+00	-.191438+00	-.186329-01	.128043-01	-.648426-01	.207130-01	.671525-02
0.2	.370243+00	-.323709+00	-.138435-01	.846244-02	-.105480+00	.314780-01	.418946-02
0.3	.510830+00	-.429566+00	-.114189-01	.652233-02	-.134960+00	.379311-01	.337755-02
0.4	.633343+00	-.512973+00	-.100267-01	.593113-02	-.152870+00	.384918-01	.357285-02
0.5	.741070+00	-.577937+00	-.933600-02	.602808-02	-.161593+00	.351345-01	.414941-02
0.6	.836640+00	-.628115+00	-.902324-02	.640812-02	-.163547+00	.296633-01	.476319-02
0.7	.922098+00	-.666411+00	-.891215-02	.687229-02	-.160653+00	.232701-01	.528566-02
0.8	.999076+00	-.695125+00	-.892017-02	.732982-02	-.154357+00	.166819-01	.566783-02
0.9	.106886+01	-.716073+00	-.896983-02	.773105-02	-.145787+00	.103423-01	.592932-02
1.0	.113248+01	-.730700+00	-.903503-02	.805665-02	-.135742+00	.447391-02	.606814-02
1.1	.119079+01	-.740170+00	-.910642-02	.830827-02	-.124857+00	-.798866-03	.611803-02
1.2	.124449+01	-.745426+00	-.915504-02	.849024-02	-.113572+00	-.546901-02	.609015-02
1.3	.129414+01	-.747231+00	-.922526-02	.861143-02	-.102227+00	-.954834-02	.601405-02
1.5	.138314+01	-.742866+00	-.929179-02	.869436-02	-.802303-01	-.161277-01	.574737-02
1.6	.142324+01	-.737641+00	-.930613-02	.867869-02	-.698554-01	-.187515-01	.558126-02
1.7	.146079+01	-.730886+00	-.931251-02	.862686-02	-.600061-01	-.210105-01	.541067-02
2.0	.156039+01	-.704327+00	-.926878-02	.834893-02	-.337065-01	-.259369-01	.490459-02
2.5	.169262+01	-.650283+00	-.905861-02	.771591-02	.487757-03	-.288676-01	.415296-02
3.0	.179507+01	-.593683+00	-.873371-02	.709259-02	.243862-01	-.277952-01	.358934-02
3.5	.187670+01	-.539416+00	-.845255-02	.651252-02	.404038-01	-.248409-01	.321518-02
4.0	.194307+01	-.489324+00	-.816670-02	.603327-02	.506675-01	-.212170-01	.296700-02
5.0	.204376+01	-.403200+00	-.765370-02	.533046-02	.600899-01	-.141746-01	.270700-02
7.0	.216853+01	-.279682+00	-.686760-02	.460480-02	.585013-01	-.472123-02	.257777-02
10.0	.226386+01	-.172868+00	-.617012-02	.422776-02	.447217-01	.697586-03	.259972-02

TABLE B.14

Load Coefficients for P_s = 2.0

(ε = 0.5, σ = 0.1, a = 0.5)

Λ	A_0	B_1	A_1	B_2	A_2	B_3	A_3
0.1	.633280-01	-.627043-01	-.255825-01	.200669-01	-.216238-01	.867326-02	.113593-01
0.2	.112569+00	-.106321+00	-.226022-01	.168130-01	-.354091-01	.118039-01	.928198-02
0.3	.156471+00	-.141747+00	-.204255-01	.146887-01	-.448862-01	.131774-01	.806391-02
0.4	.195482+00	-.170073+00	-.187823-01	.132895-01	-.507427-01	.131420-01	.736340-02
0.5	.230292+00	-.192442+00	-.176203-01	.123546-01	-.537284-01	.121271-01	.698262-02
0.7	.289471+00	-.223234+00	-.159484-01	.112190-01	-.537349-01	.842101-02	.657256-02
0.8	.314802+00	-.233295+00	-.152998-01	.108382-01	-.517755-01	.621296-02	.640983-02
0.9	.337851+00	-.240654+00	-.148078-01	.105143-01	-.490025-01	.399121-02	.626070-02
1.0	.358959+00	-.245721+00	-.144990-01	.102283-01	-.457445-01	.182187-02	.614001-02
1.1	.378257+00	-.249097+00	-.140829-01	.995071-02	-.421091-01	-.188695-03	.595876-02
1.2	.396022+00	-.251027+00	-.137130-01	.968500-02	-.382969-01	-.201880-02	.577187-02
1.3	.412474+00	-.251757+00	-.133481-01	.943262-02	-.344189-01	-.365933-02	.557274-02
1.5	.441928+00	-.250517+00	-.126981-01	.894991-02	-.267858-01	-.634655-02	.517814-02
1.6	.455207+00	-.248856+00	-.124382-01	.870768-02	-.231253-01	-.740636-01	.499388-02
1.7	.467618+00	-.246689+00	-.121350-01	.849460-02	-.196157-01	-.829635-02	.480246-02
2.0	.500598+00	-.237974+00	-.114451-01	.784970-02	-.101112-01	-.100694-01	.431383-02
2.5	.544550+00	-.219700+00	-.104061-01	.699151-02	.216240-02	-.108847-01	.366808-02
2.8	.566033+00	-.208014+00	-.992677-02	.655447-02	.761150-02	-.105907-01	.339557-02
3.0	.578762+00	-.200248+00	-.967301-02	.627817-02	.105696-01	-.102055-01	.326004-02
3.5	.606091+00	-.181483+00	-.904974-02	.574820-02	.160353-01	-.889881-02	.299414-02
4.0	.628338+00	-.164133+00	-.854135-02	.534443-02	.193921-01	-.742310-02	.282964-02
5.0	.662068+00	-.134357+00	-.775085-02	.481484-02	.221357-01	-.474384-02	.266659-02
7.0	.703647+00	-.920551-01	-.675357-02	.433880-02	.207126-01	-.144568-02	.259261-02
10.0	.735006+00	-.561763-01	-.600243-02	.412924-02	.153784-01	.226960-03	.260904-02

TABLE B.15

Load Coefficients for $P_s = 1.5$

($\epsilon = 0.5$, $\sigma = 0.1$, $a = 0.5$)

Λ	A_0	B_1	A_1	B_2	A_2	B_3	A_3
0.1	.381521-01	-.394212-01	-.266201-01	.207866-01	-.139621-01	.718020-02	.114187-01
0.2	.629091-01	-.597455-01	-.246986-01	.190897-01	-.189235-01	.701009-02	.107982-01
0.3	.855302-01	-.773153-01	-.229393-01	.172546-01	-.231823-01	.726288-02	.966170-02
0.4	.105453+00	-.910838-01	-.214731-01	.158276-01	-.255675-01	.683629-02	.880856-02
0.5	.123116+00	-.101686+00	-.202823-01	.146803-01	-.264746-01	.594562-02	.815271-02
0.8	.165492+00	-.119899+00	-.175514-01	.121962-01	-.239176-01	.219114-02	.670558-02
0.9	.176926+00	-.122815+00	-.168333-01	.115563-01	-.221107-01	.933363-03	.631811-02
1.0	.187329+00	-.124654+00	-.161828-01	.109781-01	-.200952-01	-.233629-03	.596144-02
1.1	.196843+00	-.125633+00	-.155880-01	.104518-01	-.179790-01	-.128415-02	.563311-02
1.2	.205567+00	-.125932+00	-.150335-01	.996716-02	-.158337-01	-.220721-02	.533009-02
1.3	.213646+00	-.125655+00	-.145334-01	.952833-02	-.137059-01	-.299666-02	.505195-02
1.4	.221104+00	-.124958+00	-.140547-01	.912686-02	-.116432-01	-.366997-02	.479614-02
1.5	.228041+00	-.123906+00	-.136213-01	.875078-02	-.965888-02	-.422442-02	.456560-02
1.6	.234507+00	-.122576+00	-.132199-01	.840641-02	-.777058-02	-.467181-02	.435781-02
1.7	.240551+00	-.121026+00	-.128434-01	.808817-02	-.598879-02	-.502320-02	.416931-02
2.0	.256542+00	-.115492+00	-.118613-01	.727255-02	-.131235-02	-.561180-02	.370804-02
2.5	.277736+00	-.104965+00	-.105820-01	.628663-02	.439405-02	-.556491-02	.320706-02
2.8	.288027+00	-.985647-01	-.997966-02	.585134-02	.677634-02	-.520445-02	.301878-02
3.0	.294098+00	-.943976-01	-.962181-02	.561540-02	.801330-02	-.490143-02	.292413-02
3.5	.307105+00	-.844805-01	-.894251-02	.512551-02	.101177-01	-.407545-02	.278974-02
4.0	.317574+00	-.756000-01	-.836522-02	.479962-02	.112556-01	-.324286-02	.270716-02
5.0	.333256+00	-.607807-01	-.751008-02	.442046-02	.117768-01	-.189925-02	.263527-02
7.0	.352151+00	-.406009-01	-.650025-02	.415156-02	.101872-01	-.455204-03	.260518-02
10.0	.365967+00	-.242710-01	-.579582-02	.406545-02	.719915-02	.110626-03	.262651-02

APPENDIX C

COMPUTER PROGRAMS

```
C     ******************************************************************
C     *                                                                *
C     * FINITE-DIFFERENCE FORMULATION OF THE REYNOLDS' EQUATION        *
C     * APPLIED TO AN INHERENTLY COMPENSATED LONG STRIP GAS THRUST     *
C     * BEARING.                                                       *
C     *                                                                *
C     *                                                                *
C     * THIS PROGRAM IS CAPABLE OF CALCULATING THE LOAD COEFFICIENTS   *
C     * UP TO THE THIRD HARMONICS FOR ANY GIVEN GEOMETRY(INLET LOCA-   *
C     * TION), SQUEEZE NUMBER, AMPLITUDE OF OSCILLATION, SUPPLY PRE-   *
C     * SSURE, AND RESTRICTOR COEFFICIENT. ALSO , THE PROGRAM COMPUTES.*
C     * THE AVERAGE LOAD CAPACITY,AND THE LINEAR STIFFNESS AND DAMPING *
C     *                                                                *
C     * CK       GAS CONSTANT                                          *
C     * D        DAMPING                                               *
C     * EPMAX    TOLERANCE OF CONVERGENCE                              *
C     * EPS      AMPLITUDE OF DISTURBANCE                              *
C     * II       DENOTES INLETS LOCATION IN THE COMPUTATIONAL FIELD    *
C     * IHOLE    NUMBER OF SPATIAL INCREMENTS BETWEEN THE CENTER LINE  *
C     *          AND INLETS                                            *
C     * IIHOLE   NUMBER OF SPATIAL INCREMENTS BETWEEN THE INLETS AND   *
C     *          SILL OUTER EDGE                                       *
C     * ITER     MAXIMUM NUMBER OF ITERATIONS                          *
C     * NDAT     NUMBER OF CALCULATIONS DESIRED                        *
C     * NMAX     NUMBER OF TIME STEPS                                  *
C     * OMEGN    ACCELERATION PARAMETERS WHERE N=1,2,3,4               *
C     * P(I)     PRESSURE DISTRIBUTION                                 *
C     * PO       STATIC PRESSURE DOWNSTREAM OF THE ORIFICES            *
C     * PS       SUPPLY PRESSURE                                       *
C     * REST     RESTRICTOR COEFFICIENT                                *
C     * SIG      SQUEEZE NUMBER                                        *
C     * STIFF    STIFFNESS                                             *
C     * T        TIME                                                  *
C     * TLM      TIME STEP TO START PRINTING PRESSURE ITERATIONS       *
C     * W(N)     LOAD CAPACITY DISTRIBUTION                            *
C     * WBAR     AVERAGE LOAD CAPACITY                                 *
C     *                                                                *
C     ******************************************************************
C
      IMPLICIT DOUBLE PRECISION(A-F),DOUBLE PRECISION(P,S,T,W,X,Y,R,V)
      DIMENSION P(21),PNM1(21),W(100),XK(50),WK(50),PNM2(21)
      GRIN=1.0
      TLM=7.0
      MAX=48
      DO 70 I=1,MAX
      READ(5,303)XK(I),WK(I)
   70 WRITE(6,304)XK(I),WK(I)
  303 FORMAT(2F20.14)
  304 FORMAT(' ',2E20.14)
      WRITE(6,200)
      READ(5,202)IHOLE,IIHOLE,NDAT,ITER,EPMAX,CK,NSPACE,OMEG4
      PY=3.14159265358998
      X1=0.0
      X2=2.0*PY
      NMAX=NSPACE+1
      M=IHOLE+IIHOLE
      MM2=M-2
      II=IHOLE+1
      III=IHOLE+2
      IMAX=M+1
      NS1=NMAX-2
      NS=NMAX-5
      EPMAX3=EPMAX*GRIN
      CK1=(1.-2.*CK)/CK
      CK2=(CK-1.)/CK
```

```
      CK3=(CK-1.)/(CK+1.)
      CK4=2./(CK+1.)
      CK5=1./(CK-1.)
      CK6=SQRT(CK3)
      CK7=(1.+CK)/CK
      CK8=CK6*CK4**CK5
      VALUE=CK4**(1./CK2)
      DX=1./M
      DX2=DX**2
      RDIST=IHOLE*DX
      DT=(X2-X1)/NSPACE
      TX=DT/DX2
      READ(5,201) OMEG2,OMEG3,SIG,EPS
      WRITE(6,203)SIG,EPS,RDIST,DX,CK,VALUE,IMAX,NSPACE,ITER,EPMAX,EPMAX
     13,X1,X2
      WRITE(6,218) OMEG2,OMEG3
      DO 2000 JJ=1,NDAT
      READ(5,204) REST,PS,PO,OMEG1
      WRITE(6,205) REST,PS,PO,OMEG1
C
      PSCK=PS**(1./CK)
      PSCK2=PS**CK2
C
C     *****  INITIAL CONDITIONS  *****
C     *****        REGION I      *****
C
      DO 21 I=1,IHOLE
   21 P(I)=PO
C     *****        REGION II     *****
      DO 22 I=I1,M
      X=DY*(I-1)
   22 P(I)=DSQRT(1.0+(PO**2-1.0)*(1.0-X)/(1.0-RDIST))
      P(IMAX)=1.0
      WRITE(6,213)(P(I),I=1,IMAX)
C
C     *****  STATIC LOAD CAPACITY   *****
C
      W(1)=(PO**3-1.0)*(1.0-RDIST)*2.0/(3.0*(PO**2-1.0))+RDIST*PO-1.0
C
      DO 23 I=1,IMAX
   23 PNM1(I)=P(I)
C
C     *****  START TIME ITERATION  *****
C
      T=0.0
      DO 1000 N=2,NMAX
C
C     *****  SET UP TIME INCREMENT   *****
C            ACCORDING TO THE ABSCISSAS
C            AND WEIGHT FACTORS FOR
C            GAUSSIAN INTEGRATION
C
      IF(N.EQ.NMAX) GO TO 71
      YK=PY*(1.0+XK(N-1))
      DT=YK-T
      T=YK
      IF(N.EQ.2) GO TO 95
      IF(N.EQ.3) GO TO 96
      TB=PY*(1.0+XK(N-2))
      TBB=PY*(1.0+XK(N-3))
      GO TO 72
C
   95 TB=0.0
      TBB=0.0
      GO TO 72
```

```
   96 TB=PY*(1.0+XK(N-2))
      TBB=0.0
      GO TO 72
   71 CONTINUE
      DT=X2-T
      T=X2
      TB=PY*(1.0+XK(N-2))
      TBB=PY*(1.0+XK(N-3))
   72 CONTINUE
C
      IF(T.GE.TLM) WRITE(6,211) T
      CT=DCOS(T)
      ST=DSIN(T)
      STA=DSIN(TB)
      STAA=DSIN(TBB)
      FAA=1.0+EPS*STA
      FAB=1.+EPS*ST
      FBB=1.0+EPS*STAA
      R=FAB**2*TX/SIG
      RS=FAA/FAB
      RSS=FBB/FAB
      F=(REST*PS**2)/(FAB**2*(1.-RDIST))
      F1=2.*DX*F*CK8/PS
C
C     *****  START OUTER ITERATION  *****
C
      O1=OMEG1
      DO 1 KK=1,ITER
C
C     *****  RESET OUTER ITERATION  *****
C            AVERAGING PARAMETER TO
C            THE DESIRED VALUE AFTER
C            FIRST ITERATION
C
      IF(KK.EQ.1) OMEG1=1.0
      IF(KK.GT.1) OMEG1=O1
C
C     *****  CHECK MASS FLOW DIRECTION  *****
C
      PIII=P(II)
      TRY1=PIII-PS
      IF(TRY1)9,9,99
C
C     *****  FOWARD CRITICAL FLOW EQUATION  *****
C            GIVES INLET PRESSURE EXPLICITLY
C
    9 PI=DSQRT(2.*(P(II+1)**2+P(II-1)**2)/3.-(P(II+2)**2+P(II-2)**2)/6.+
     $DX*F*CK8/3.)
C
C     *****  CHECK MASS FLOW INTENSITY  *****
C
      TRY2=PI/PS
      IF(TRY2.GT.VALUE.AND.TRY2.LT.(VALUE+0.05)) GO TO 809
      IF(TRY2-VALUE)809,809,810
  810 CONTINUE
      PLAST=P(II)
      P(II)=PLAST
      GO TO 80
  809 PRHS=F*CK8
      IF(T.GE.TLM) WRITE(6,215) PI,PRHS
      ITEMP1=0
      GO TO 12
C
C     *****  FOWARD SUBCRITICAL FLOW EQUATION  *****
C            GIVES INLET PRESSURE IMPLICITLY
```

```
C            AND IT IS ITERATIVELY SOLVED
C
   80 CONTINUE
      DO 100 K1=1,ITER
      IF(P(II).GT.PS) GO TO 880
    8 PI=DSQRT(2.*(P(II+1)**2+P(II-1)**2)/3.-(P(II+2)**2+P(II-2)**2)/6.+
     $DX*F*P(II)**(1./CK)*(1.-P(II)**CK2/PSCK2)**.5/(PSCK*3.))
      PI=OMEG4*PI+(1.0-OMEG4)*P(II)
      TRY5=PI-PS
      PA=P(II)
      IF(T.GE.TLM) WRITE(6,504) PI,PA
  504 FORMAT(' ',' F',2E14.8)
      IF(PI.GE.(PS-.02).AND.PI.LE.(PS+.05)) GO TO 12
C
      IF(TRY5)16,16,93
C
   93 CONTINUE
      P(II)=PI
      GO TO 868
   16 CONTINUE
      TRY7=PI/PS
      IF(TRY7-VALUE)9,9,91
   91 CONTINUE
      PERE=DABS((PI-P(II))/P(II))
      P(II)=PI
      IF(PERE.LT.EPMAX) GO TO 808
  100 CONTINUE
C
C     *****  ERROR MESSAGE  *****
C
      GO TO 998
  808 PRHS=F*PI**(1.0/CK)*(1.-PI**CK2/PSCK2)**.5/PSCK
      IF(T.GE.TLM) WRITE(6,214) PI,PRHS
      ITEMP1=K1
      GO TO 12
C
C     *****  BACKWARD CRITICAL FLOW EQUATION  *****
C            GIVES INLET PRESSURE EXPLICITLY
C
   99 PI=(DSQRT(F1**2+24.*(4.*(P(II+1)**2+P(II-1)**2)-(P(II+2)**2+P(II-2
     $)**2)))-F1)/12.
C
C     *****  CHECK MASS FLOW INTENSITY  *****
C
      TRY3=PS/PI
      IF(TRY3-VALUE)899,899,900
  900 CONTINUE
      PLAST=P(II)
      P(II)=PLAST
      GO TO 880
  899 PRHS=F*PI*CK8/PS
      IF(T.GE.TLM) WRITE(6,217) PI,PRHS
      ITEMP1=0
      GO TO 12
C
C     *****  BACKWARD SUBCRITICAL FLOW EQUATION  *****
C            GIVES INLET PRESSURE IMPLICITLY
C            AND IT IS ITERATIVELY SOLVED
C
  880 CONTINUE
      DO 101 K1=1,ITER
      IF(P(II).LE.PS) GO TO 80
   88 PI=DSQRT(2.*(P(II+1)**2+P(II-1)**2)/3.-(P(II+2)**2+P(II-2)**2)/6.-
     $DX*F*(1.-PSCK2/P(II)**CK2)**.5*P(II)**CK2/(PSCK2*3.))
```

```
      PI=OMEG4*PI+(1.0-OMEG4)*P(II)
      TRY6=PI-PS
      PA=P(II)
      IF(T.GE.TLM) WRITE(6,505) PI,PA
  505 FORMAT(' ',' B',2E14.8)
      IF(PI.LE.(PS+.02).AND.PI.GE.(PS-.05)) GO TO 12
C
      IF(TRY6)94,94,18
C
   94 CONTINUE
      P(II)=PI
      GO TO 808
   18 CONTINUE
      TRY8=PS/PI
      IF(TRY8-VALUE)99,99,92
   92 CONTINUE
      PERE=DABS((PI-P(II))/P(II))
      P(II)=PI
      IF(PERE.LT.EPMAX) GO TO 888
  101 CONTINUE
C
C     *****  ERROR MESSAGE  *****
C
      GO TO 998
  888 PRHS=F*(1.0-PSCK2/PI**CK2)**.5*PI**CK2/PSCK2
      IF(T.GE.TLM) WRITE(6,216) PI,PRHS
      ITEMP1=K1
C
C     *****  AVERAGE OF CURRENT INLET  *****
C            PRESSURE VALUE WITH PREVIOUS
C            ONE ( OUTER ITERATION )
C
   12 PII=PI
      PII=OMEG1*PII+(1.-OMEG1)*PIII
      PERR=DABS(PII-PIII)
      P(II)=PII
      IF(II.LT.3) GO TO 5
C
C     *****  START POINT S.O.R. ITERATION  *****
C            TO OBTAIN PRESSURE DISTRIBUTION
C            IN REGION I
C
      DO 3 K=1,ITER
      PERI=0.0
      DO 2 J=1,IHOLE
      I=IHOLE-(J-1)
      IF(I.NE.1) GO TO 11
      PI=(4.*P(I+1)-P(I+2))/3.
      GO TO 10
   11 CONTINUE
      IF(T-DT)13,13,15
   13 PI=(0.5*R*(P(I+1)**2+P(I-1)**2)+RS*PNM1(I))/(1.0+R*P(I))
      GO TO 10
   15 PI=(R*(P(I+1)**2+P(I-1)**2)+4.0*RS*PNM1(I)-RSS*PNM2(I))/(3.0+2.0*R
     $*P(I))
C
C     *****  ACCELERATION PARAMETER EQUATION  *****
C
   10 PI=OMEG2*PI+(1.-OMEG2)*P(I)
      TEMP=P(I)
      CALL ERROR(TEMP,PI,PERI,ANS)
      PERI=ANS
      P(I)=PI
    2 CONTINUE
      IF(PERI.LT.EPMAX) GO TO 4
```

```
    3 CONTINUE
C
C       *****  ERROR MESSAGE  *****
C
        WRITE(6,206)ITER
    4 ITEMP=K
C
C       *****  START POINT S.O.R. ITERATION  *****
C              TO OBTAIN PRESSURE DISTRIBUTION
C              IN REGION II
C
    5 DO 7 K=1,ITER
      PERII=0.0
      DO 6 I=III,M
      IF(T-DT)17,17,19
   17 PI=(0.5*R*(P(I+1)**2+P(I-1)**2)+RS*PNM1(I))/(1.0+R*P(I))
      GO TO 20
   19 PI=(R*(P(I+1)**2+P(I-1)**2)+4.0*RS*PNM1(I)-RSS*PNM2(I))/(3.0+2.0*R
     5*P(I))
C
C       *****  ACCELERATION PARAMETER EQUATION  *****
C
   20 PI=OMEG3*PI+(1.-OMEG3)*P(I)
      TEMP=P(I)
      CALL ERROR(TEMP,PI,PERII,ANS)
      PERII=ANS
      P(I)=PI
    6 CONTINUE
      IF(PERII.LT.EPMAX) GO TO 44
    7 CONTINUE
C
C       *****  ERROR MESSAGE  *****
C
      WRITE(6,207)ITER
   44 IF(T.GE.TLM) WRITE(6,401) ITEMP,K,ITEMP1
  401 FORMAT(' ',' I',I3,3X,'II',I3,3X,'NE',I3)
      IF(T.GE.TLM) WRITE(6,209) (P(I),I=1,IMAX)
      IF(PERR.LT.EPMAX3) GO TO 444
    1 CONTINUE
C
C       *****  ERROR MESSAGE  *****
C
      WRITE(6,210)ITER
      GO TO 2000
  444 IF(T.GE.TLM) WRITE(6,402) KK
  402 FORMAT(' ',' OUTER',I5)
C
C       *****  CALCULATE LOAD CAPACITY  *****
C              AT EACH TIME STEP
C
      SUMWX=0.0
      DO 31 IN=2,M,2
      SUMWX=SUMWX+(P(IN-1)+4.*P(IN)+P(IN+1))
   31 CONTINUE
      W(N)=SUMWX*DX/3.-1.0
      WN=W(N)
      IF(T.GE.TLM) WRITE(6,302) WN
C
C       *****  RESET PREVIOUS TIME  *****
C              PRESSURE DISTRIBUTION
C              WITH CURRENT ONE
C
      DO 55 I=1,IMAX
      PNM2(I)=PNM1(I)
   55 PNM1(I)=P(I)
```

```
 1000 CONTINUE
C
C      *****  CALCULATE LOAD COEFFICIENTS  *****
C
       CALL GAUSS(XK,WK,W,1.0,1,MAX,NMAX,ANSB1)
       B1=ANSB1/PY
C
C      *****  LINEAR STIFFNESS  *****
C
       STIFF=-1.0*B1/((PS-1.0)*EPS)
       CALL GAUSS(XK,WK,W,2.0,1,MAX,NMAX,ANSB2)
       B2=ANSB2/PY
       CALL GAUSS(XK,WK,W,3.0,1,MAX,NMAX,ANSB3)
       B3=ANSB3/PY
       CALL GAUSS(XK,WK,W,1.0,2,MAX,NMAX,ANSA1)
       A1=ANSA1/PY
C
C      *****  LINEAR DAMPING  ****
C
       DAMP=-12.0*A1*(1.0-EPS**2)**1.5/(SIG*EPS)
       CALL GAUSS(XK,WK,W,2.0,2,MAX,NMAX,ANSA2)
       A2=ANSA2/PY
       CALL GAUSS(XK,WK,W,3.0,2,MAX,NMAX,ANSA3)
       A3=ANSA3/PY
       CALL GAUSS(XK,WK,W,0.0,3,MAX,NMAX,ANSWB)
       A0=ANSWB/(2.0*PY)
C
C      *****  AVERAGE LOAD CAPACITY  *****
C
       WBAR=A0/(PS-1.0)
C
       WRITE(6,220)  A0,B1,A1,B2,A2,B3,A3
       WRITE(6,221)  WBAR,STIFF,DAMP
 2000 CONTINUE
       GO TO 999
C
  998 WRITE(6,675)
  999 STOP
  200 FORMAT('1 THIS PROGRAM EVALUATES THE AVERAGE LOAD CAPACITY,STIFFNE
     1SS AND DAMPING FOR AN INHERENTLY COMPENSATED LONG STRIP GAS BEARIN
     2G')
  201 FORMAT(5F10.5)
  202 FORMAT(4I5,F10.6,F10.4,I5,F10.4)
  203 FORMAT(//,' ',F10.5,F6.3,F10.5,E13.5,F10.5,E11.5,3I5,2E7.1,2F10.5,
     1//)
  204 FORMAT(2F10.3,2F10.7)
  205 FORMAT(/' REST =',F5.2,3X,'PS =',F5.2,3X,'PO =',F9.5,5X,'OMEG1 =',
     1F8.3,/)
  206 FORMAT(' ',' NUMBER OF ITERATIONS HAS BEEN EXCEEDED IN REGION I  I
     $TER= ',I4)
  207 FORMAT(' ',' NUMBER OF ITERATIONS HAS BEEN EXCEEDED IN REGION II I
     $TER= ',I4)
  209 FORMAT(' ',11E11.5)
  210 FORMAT(' ',' NUMBER OF ITERATIONS HAS BEEN EXCEEDED IN OUTER ITERA
     $TIVE PROCEDURE  ITER= ',I4,///)
  211 FORMAT(/' ',' TIME = ',E11.5)
  213 FORMAT(/' ',11E11.5)
  214 FORMAT(' ',' FOWARD SUBCRITICAL FLOW',2E14.8)
  215 FORMAT(' ',' FOWARD CRITICAL FLOW',2E14.8)
  216 FORMAT(' ',' BACKWARD SUBCRITICAL FLOW',2E14.8)
  217 FORMAT(' ',' BACKWARD CRITICAL FLOW',2E14.8)
  218 FORMAT(/,' OMEG2 =',F8.3,3X,'OMEG3 =',F8.3,/)
  220 FORMAT(//' ',' A0 =',E11.6,3X,'B1 =',E11.6,3X,'A1 =',E11.6,3X,'B2
     1 =',E11.6,3X,'A2 =',E11.6,3X,'B3 =',E11.6,3X,'A3 =',E11.6)
  221 FORMAT(/,' ',' WBAR =',E11.6,3X,'STIFF =',E11.6,3X,'DAMP =',E11.6,
```

```
     1//)
302 FORMAT(' ',E11.5)
675 FORMAT(//,' ',' CHECK INLET ITERATIVE EQUATION, IT DOES NOT CONVER
     1GE')
      END
```

```
C     ******************************************************************
C     *                                                                *
C     * THIS SUBROUTINE CHECKS THE ABSOLUTE OR RELATIVE ERROR BETWEEN  *
C     * A CURRENT PRESSURE VALUE AND THE PREVIOUS ONE FOR THE ITERATI- *
C     * VE PROCEDURES UTILIZED IN THE MAIN PROGRAM.                    *
C     *                                                                *
C     ******************************************************************
C
      SUBROUTINE ERROR(X1,X2,ERRI,ERRO)
      IMPLICIT DOUBLE PRECISION(E,T,X)
      TEST=DABS(X2-X1)
      IF(X2.GT.1.0) GO TO 1
      IF(TEST.GT.ERRI) ERRI=TEST
      GO TO 2
    1 IF((TEST/X1).GT.ERRI) ERRI=TEST/X1
    2 ERRO=ERRI
      RETURN
      END
```

```
C     ******************************************************************
C     *                                                                *
C     * THIS SUBROUTINE CALCULATES THE FOURIER COEFFICIENTS UP TO ANY  *
C     * DESIRED HARMONIC ORDER.  THE ABSCISSA AND WEIGHT FACTORS FOR   *
C     * GAUSSIAN INTEGRATION MUST BE PROVIDED AS DATA IN THE MAIN      *
C     * PROGRAM AND THEY CAN BE OBTAINED FROM ANY STANDARD REFERENCE   *
C     *                                                                *
C     * XK      ABSCISSAS                                              *
C     * WK      WEIGHTS                                                *
C     * R       ORDER OF HARMONIC                                      *
C     * MAX     NUMBER OF ABSCISSAS                                    *
C     * KEY     1 - SINE TERM, 3 - COSINE TERM, 5 - AVERAGE TERM       *
C     * NMAX    NUMBER OF TIME STEPS INCLUDING T=0                     *
C     *                                                                *
C     ******************************************************************
C
      SUBROUTINE GAUSS(XK,WK,W,R,KEY,MAX,NMAX,TOTAL)
      IMPLICIT DOUBLE PRECISION(A,P,S,T,W,Y,X)
      DIMENSION XK(MAX),WK(MAX),W(NMAX)
      PY=3.1415926535898
      SUM=0.0
      GO TO (1,3,5),KEY
    1 DO 2 I=1,MAX
      YK=PY*(1.0+XK(I))
      SUM=SUM+W(I+1)*WK(I)*DSIN(R*YK)
    2 CONTINUE
      GO TO 7
    3 DO 4 I=1,MAX
      YK=PY*(1.0+XK(I))
      SUM=SUM+W(I+1)*WK(I)*DCOS(R*YK)
    4 CONTINUE
      GO TO 7
    5 DO 6 I=1,MAX
      YK=PY*(1.0+XK(I))
      SUM=SUM+W(I+1)*WK(I)
    6 CONTINUE
    7 TOTAL=PY*SUM
      RETURN
      END
```

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